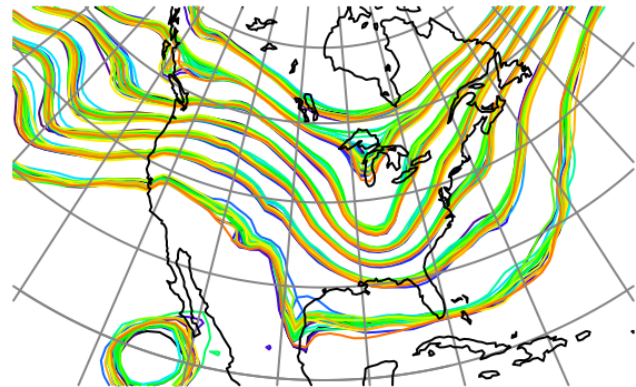




Intro to DA: The Ensemble Kalman Filter

Jeff Anderson representing the NCAR Data Assimilation Research Section



©UCAR 2014

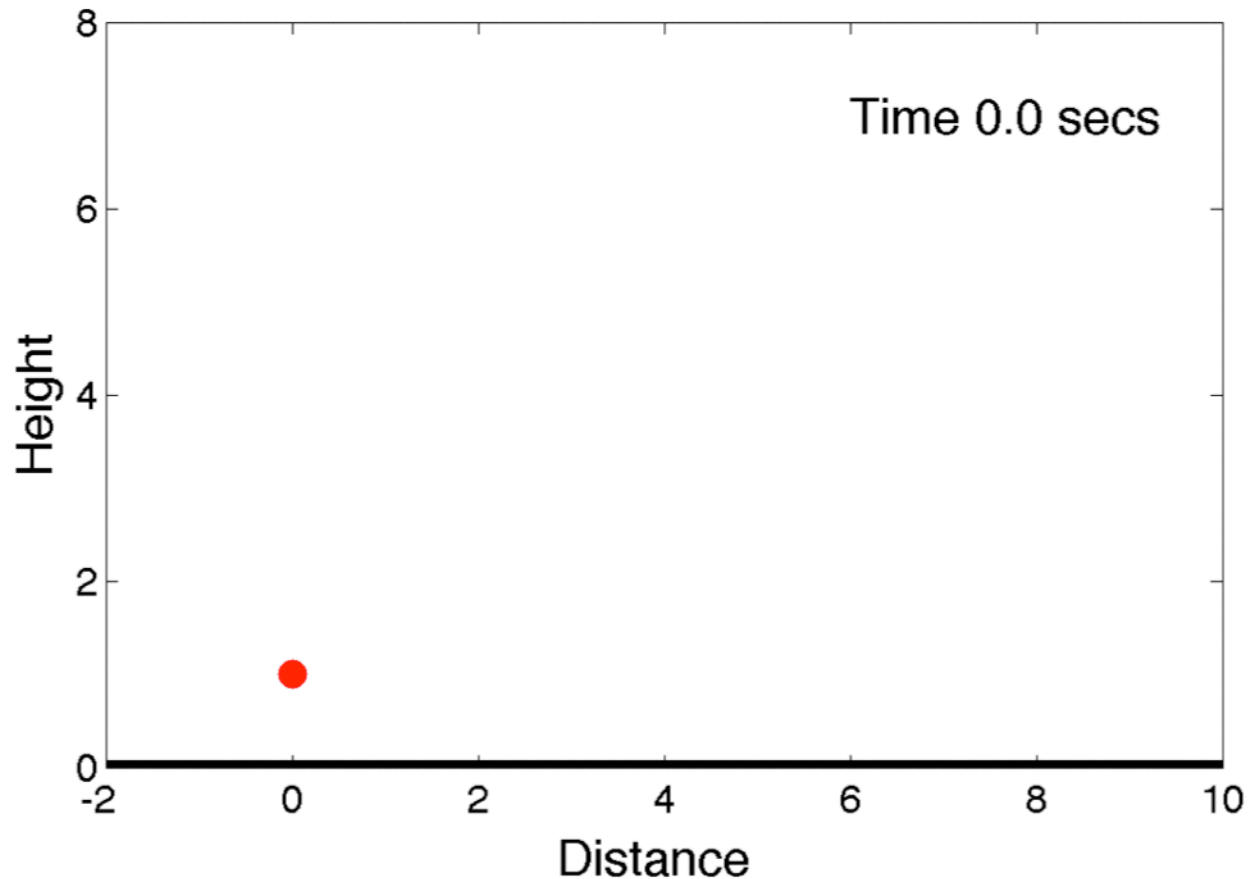


The National Center for Atmospheric Research is sponsored by the National Science Foundation. Any opinions, findings and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

NCAR | National Center for
UCAR | Atmospheric Research

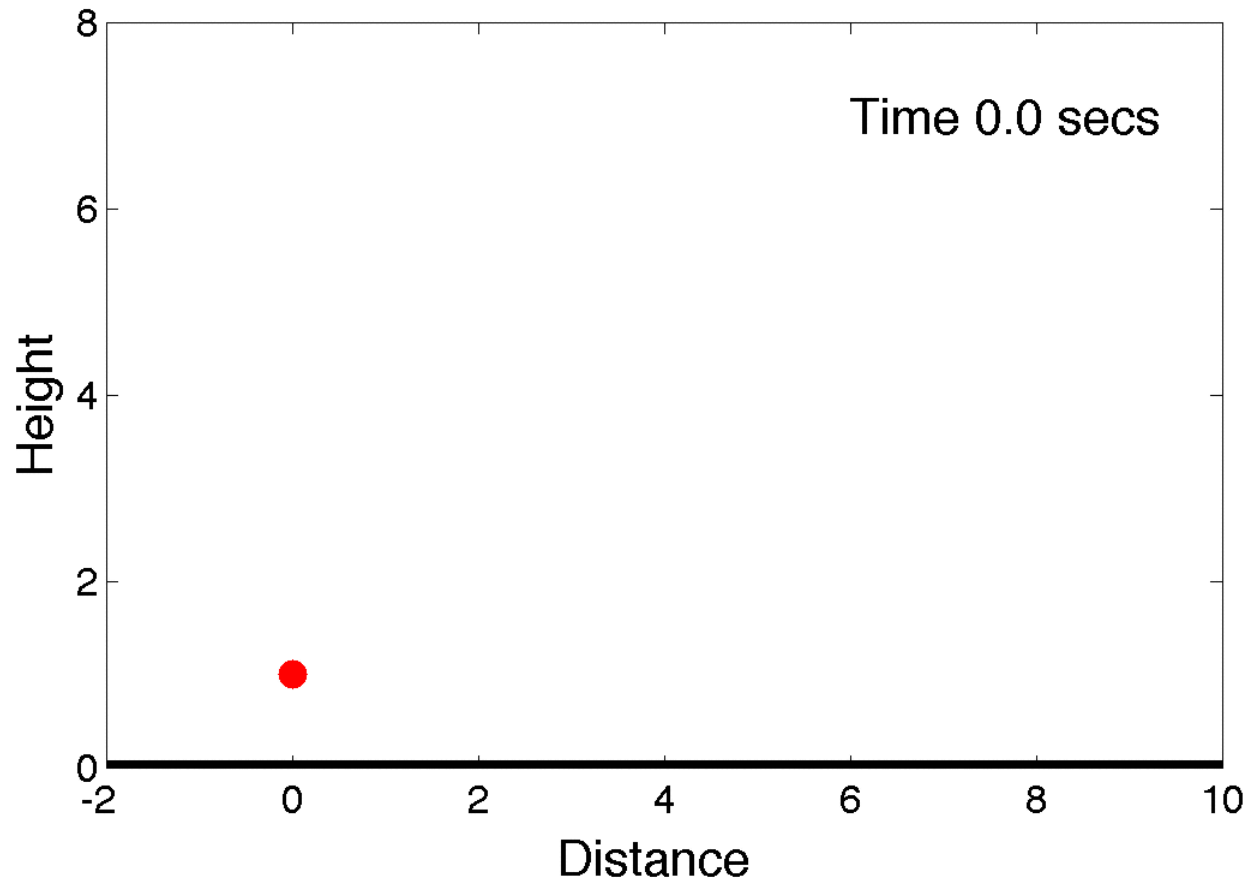
Building a Forecast System

Want to predict where the ball will land.



Building a Forecast System

Want to predict where the ball will land.



Building a Forecast System

Prediction Model

Building a Forecast System

Prediction Model

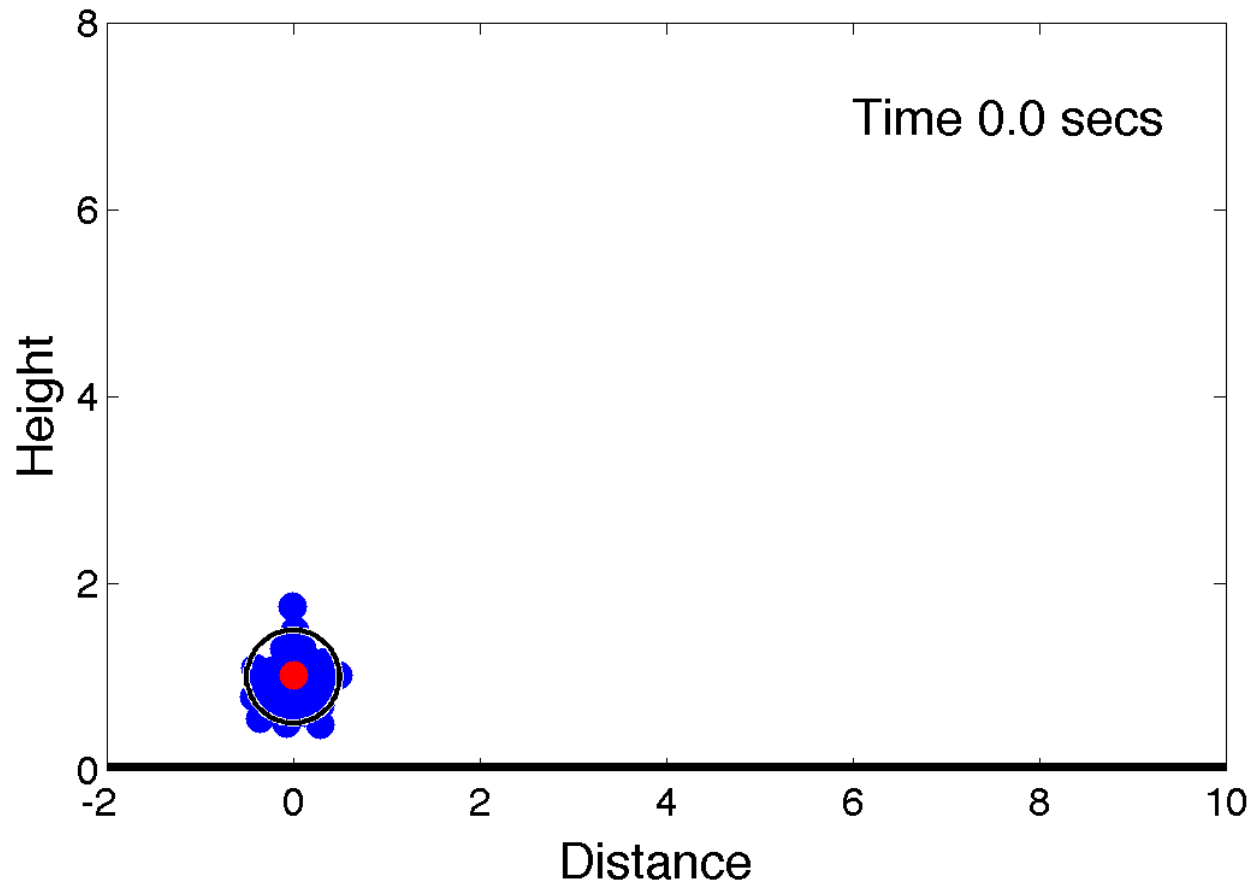
For the ball this is simple:

$$x = x_{initial} + u_{initial}t$$

$$y = y_{initial} + v_{initial}t - 1/2 gt^2$$

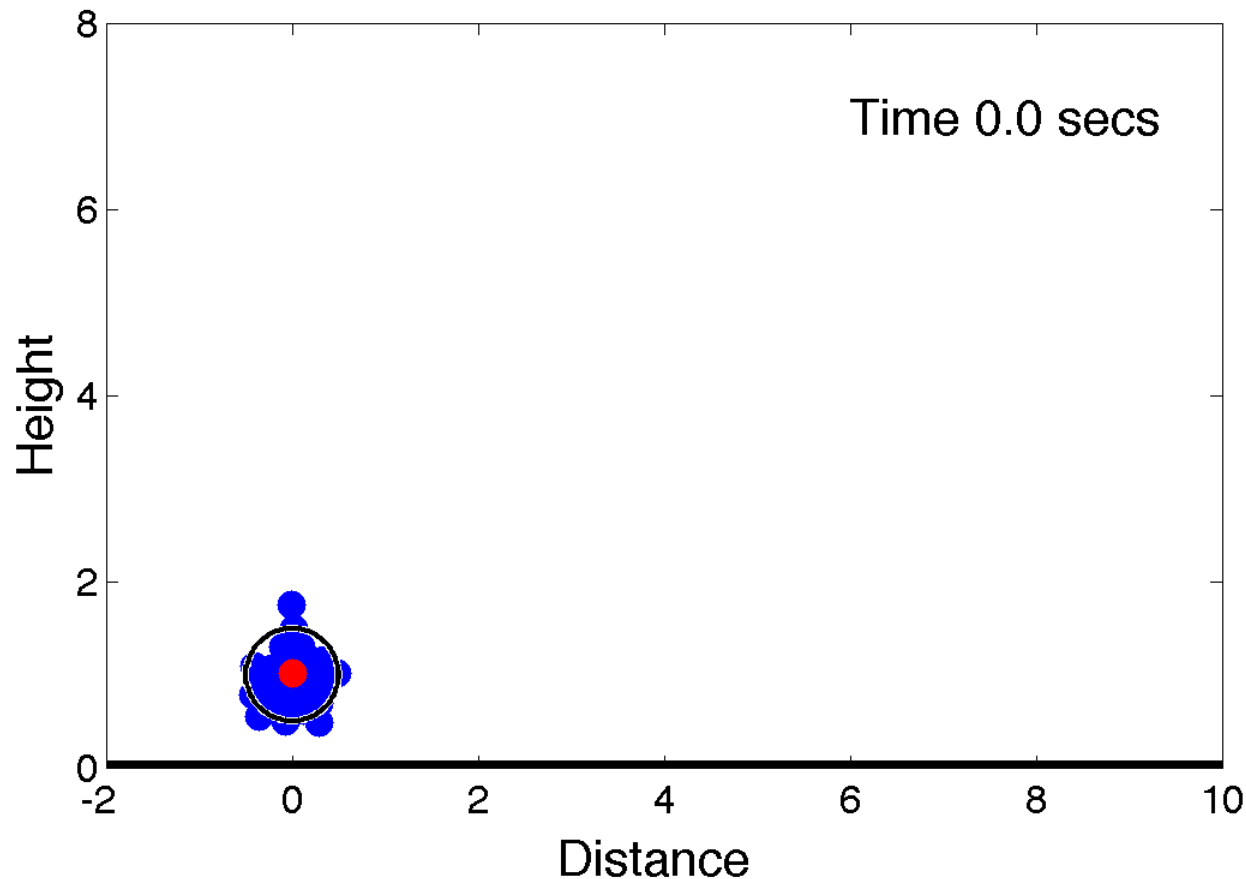
An Ensemble of Model Forecasts Shows Uncertainty

Unsure about release point, velocity, angle...
Sample this with an 'ensemble' of blue balls.



An Ensemble of Model Forecasts Shows Uncertainty

Unsure about release point, velocity, angle...
Sample this with an 'ensemble' of blue balls.



Building a Forecast System

Prediction Model

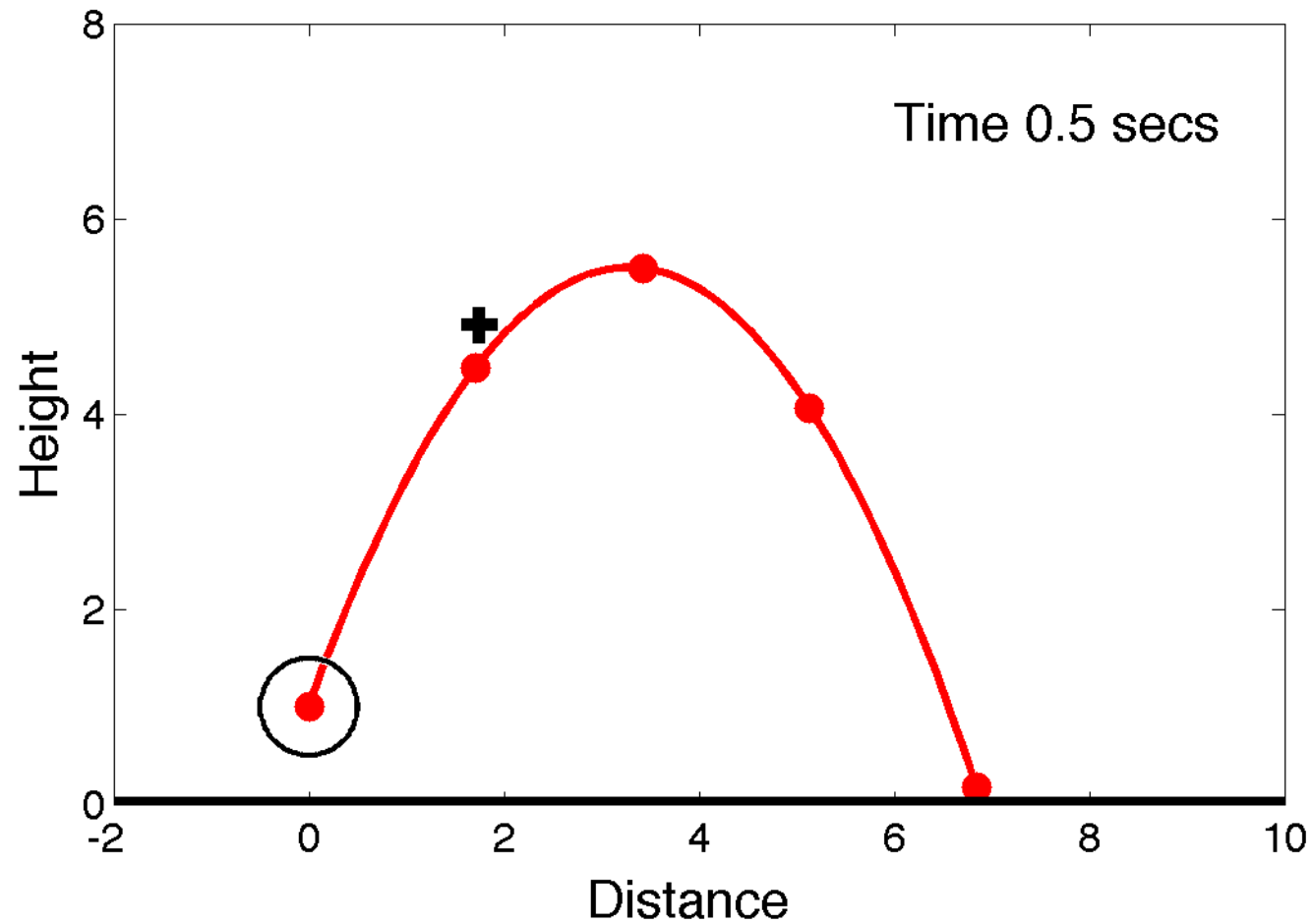
Observing System

Need observations (measurements) of the red ball.

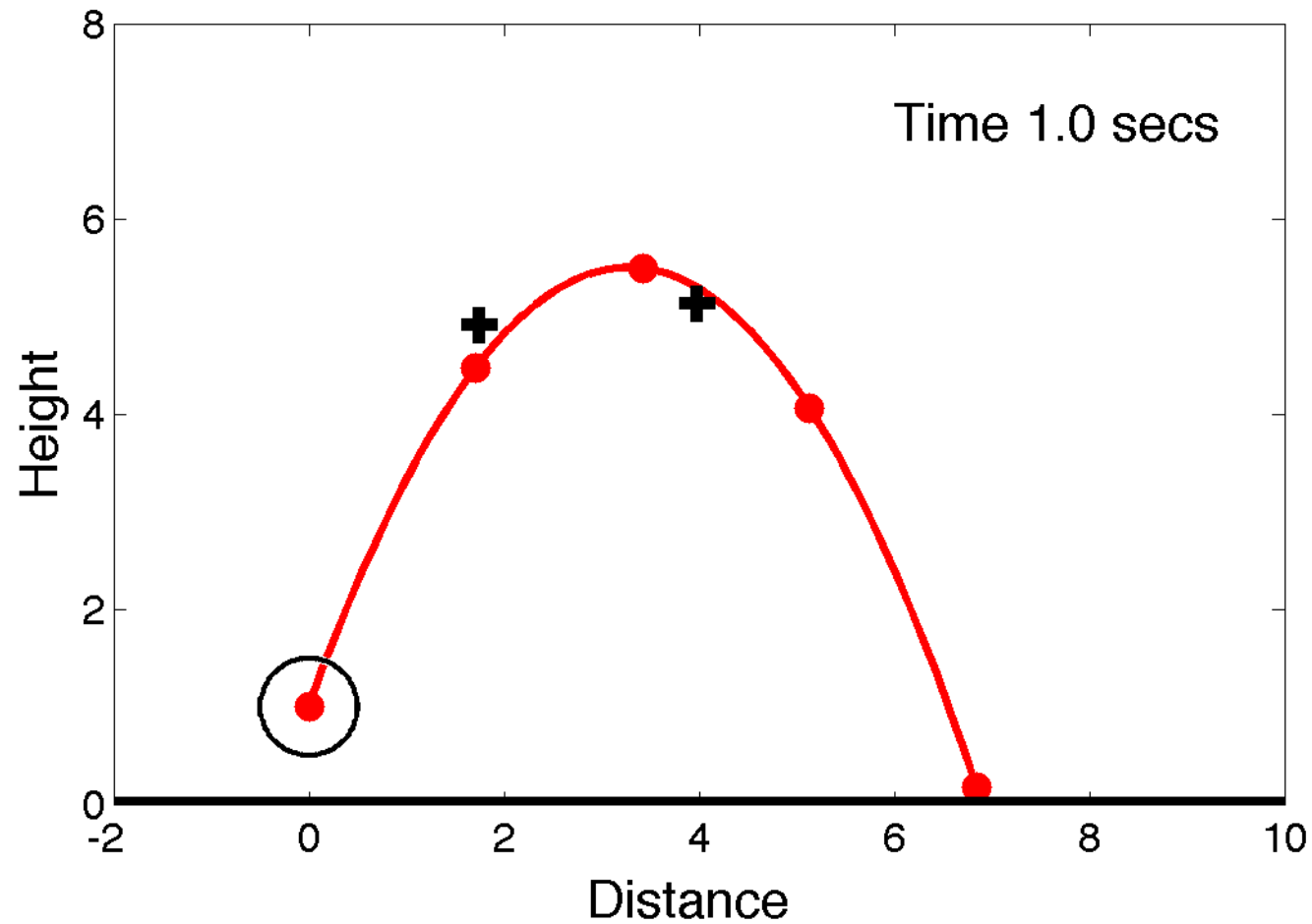
All observations have errors.

Observe position of ball every half second after throw.

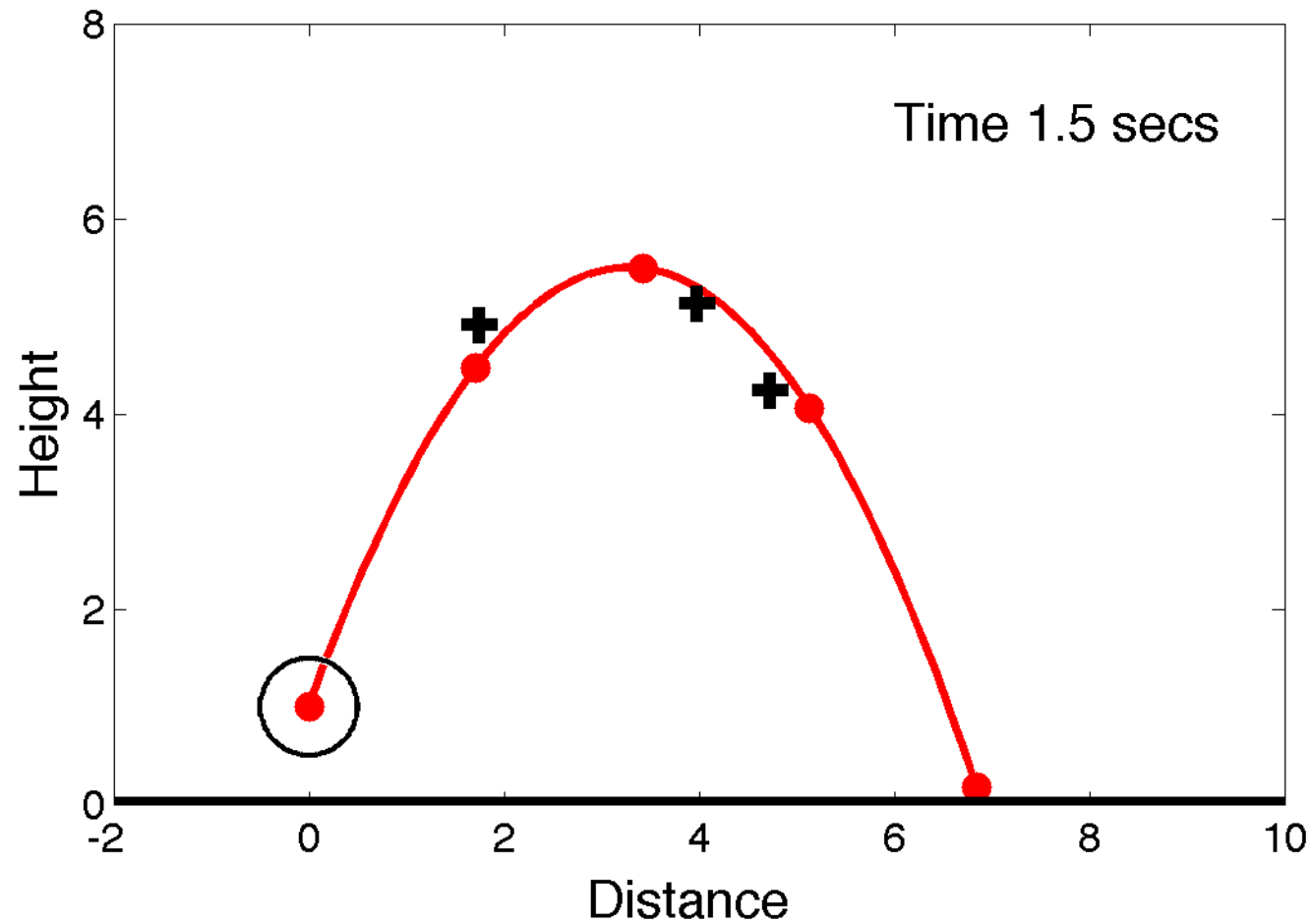
Observations of the Red Ball



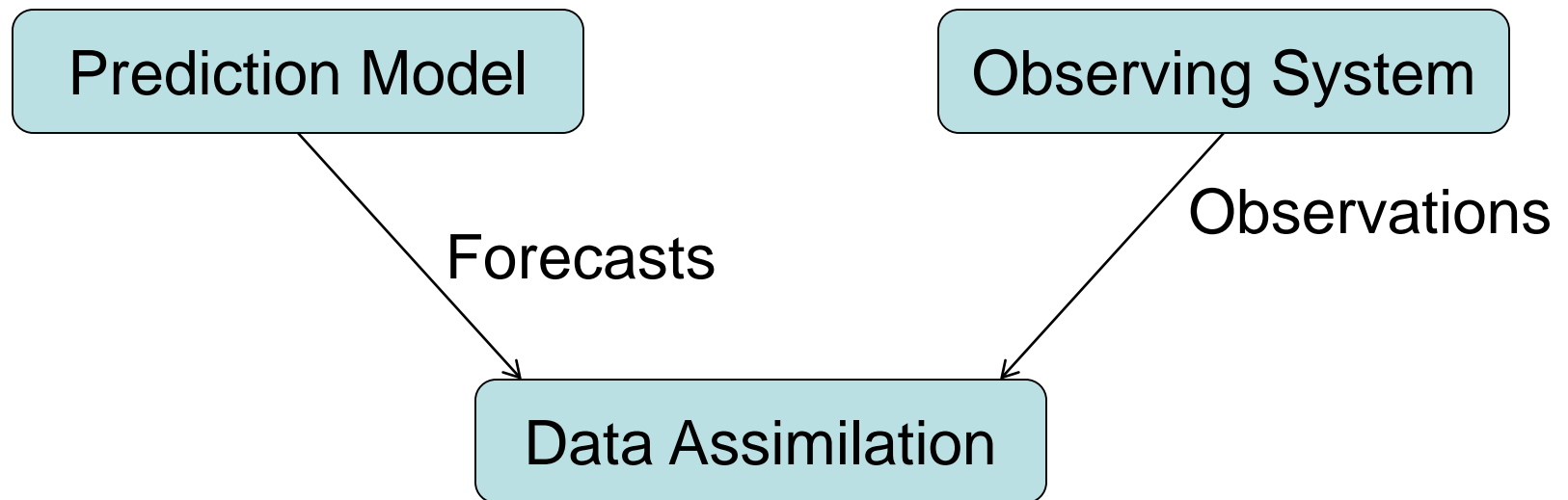
Observations of the Red Ball



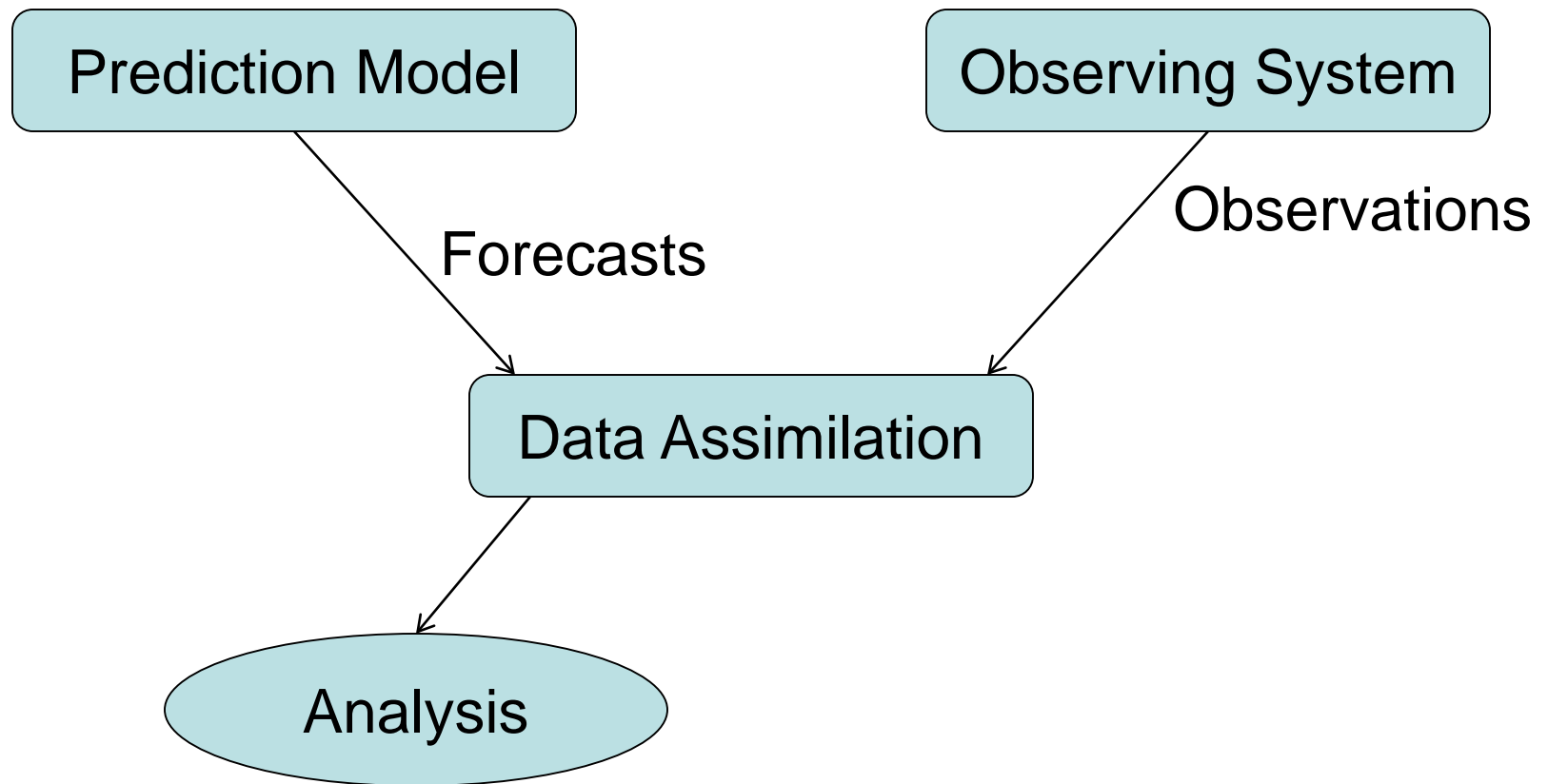
Observations of the Red Ball



Building a Forecast System



Building a Forecast System

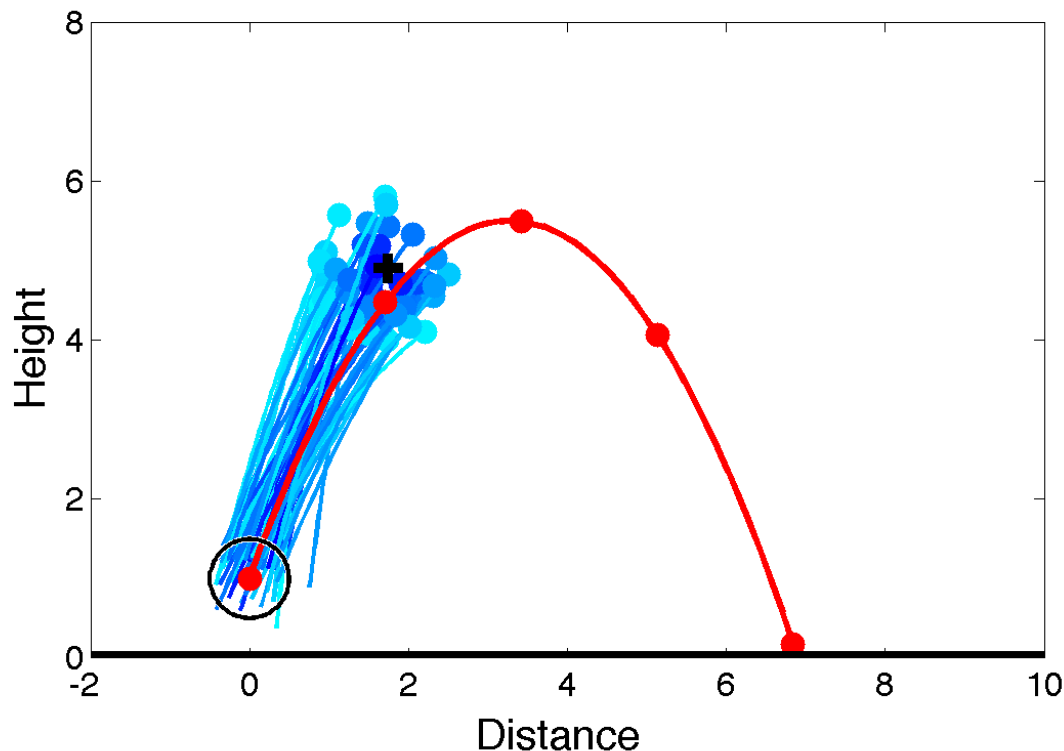


Assimilating the First Observation

Make large ensemble of forecasts.

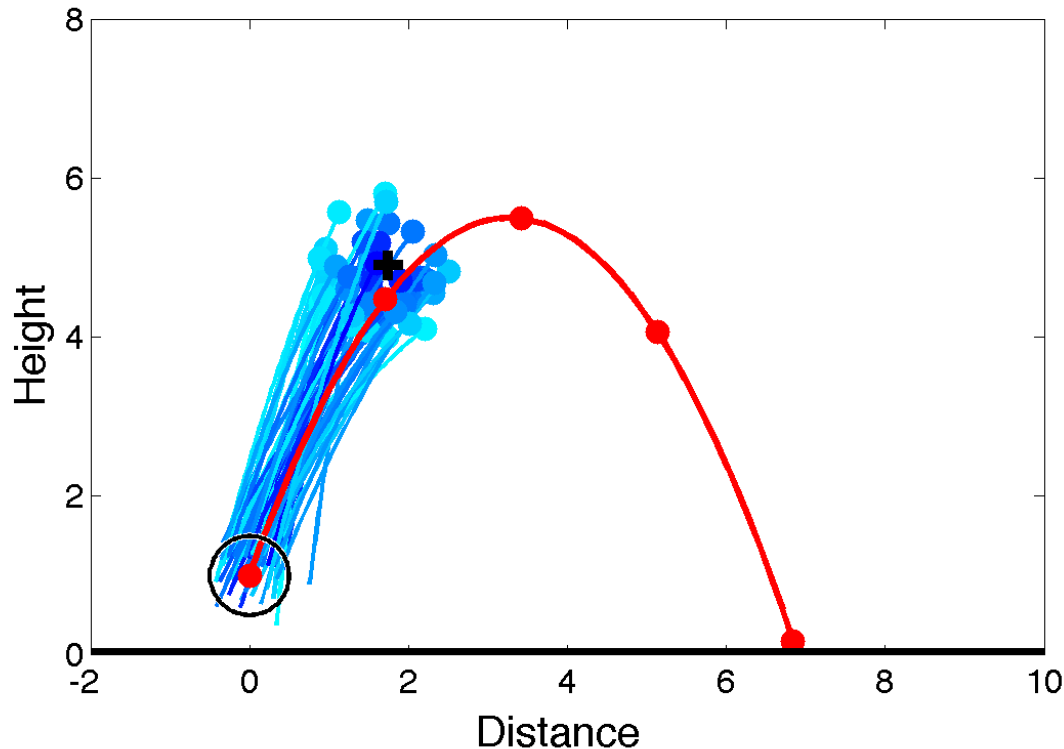
Closer to observation => more likely.

Fifty likely forecasts are shown (darker blue => more likely).

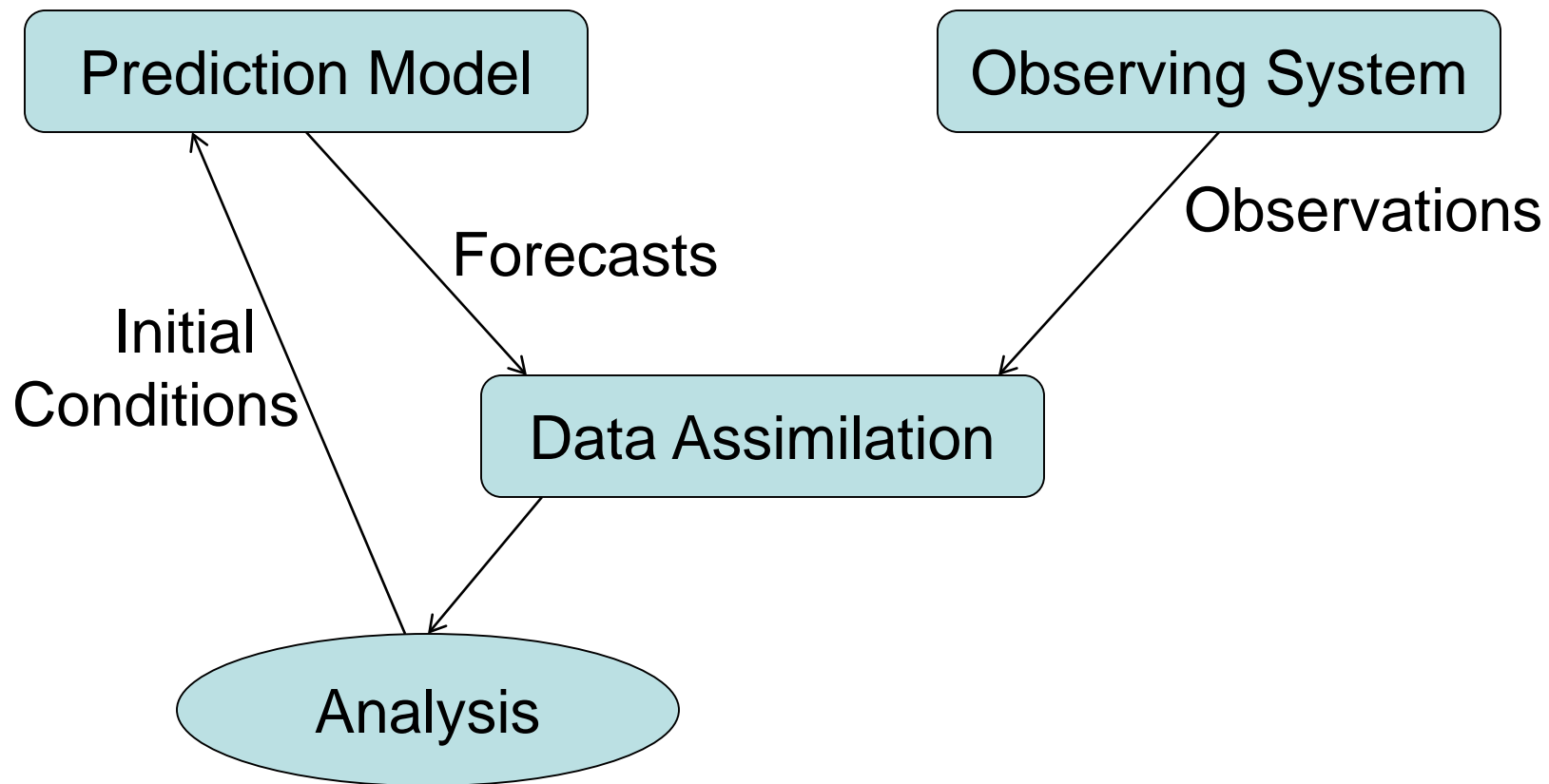


Assimilating the First Observation

Fifty balls at time 0.5 are an ensemble analysis.
Show uncertainty of best estimate of red ball's location.

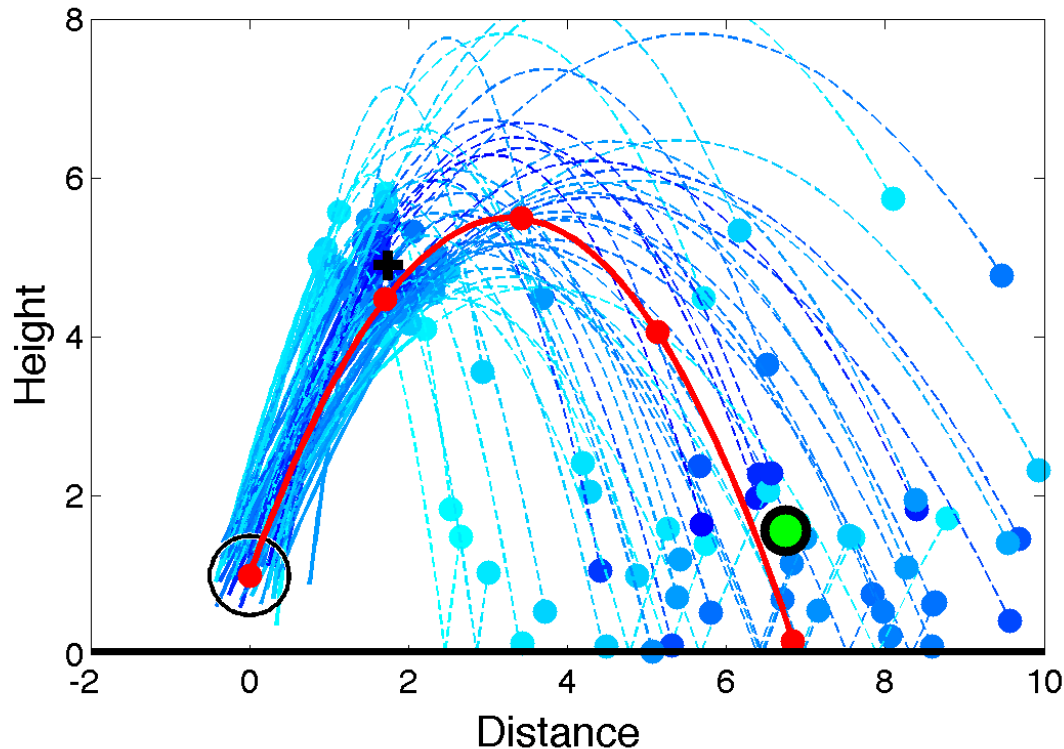


Building a Forecast System

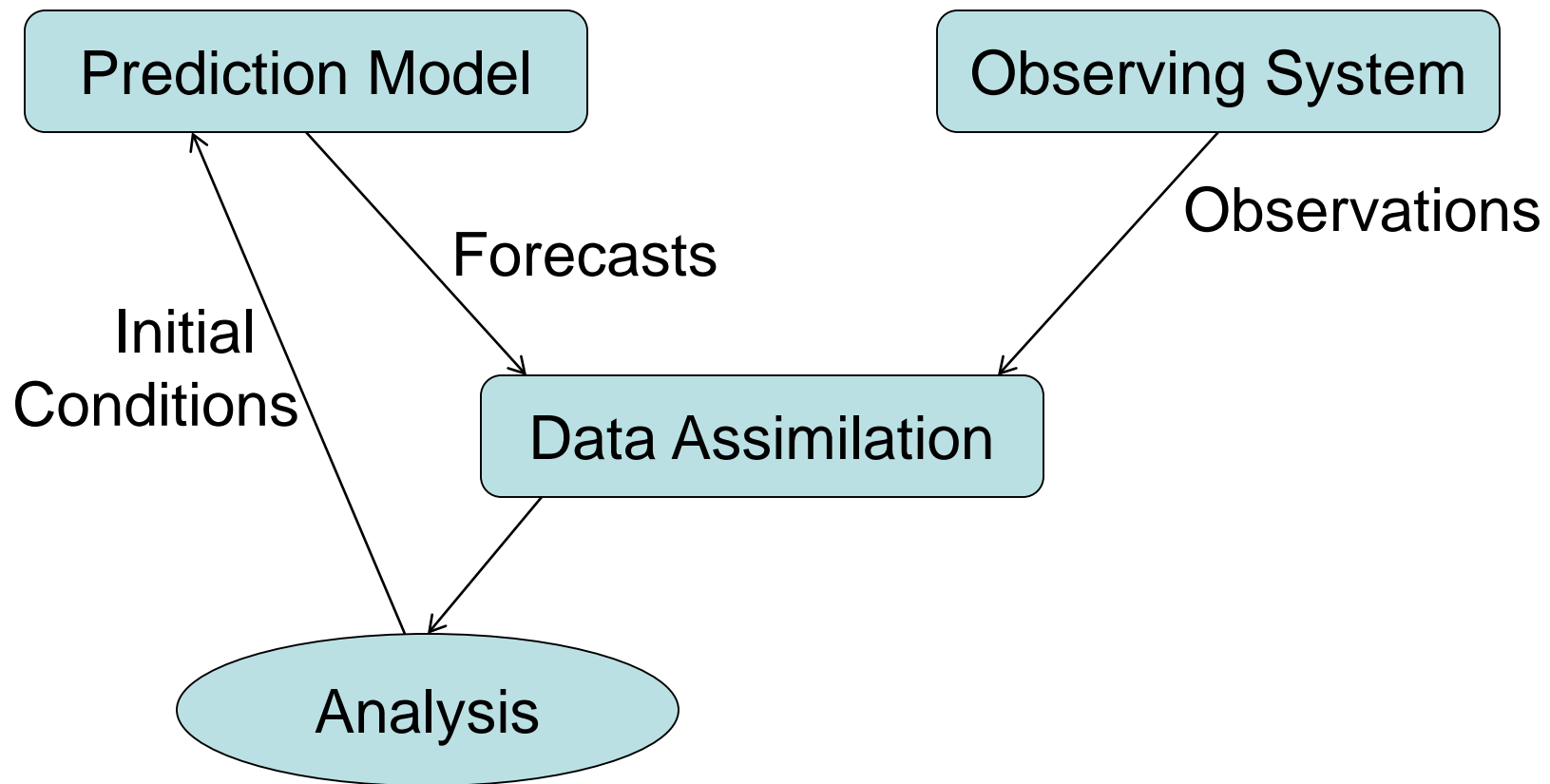


Assimilating the First Observation

Analysis ensemble are initial conditions for 50 forecasts.
Green is weighted mean of ensemble forecast at time 2.0.
This is best single forecast given observations at time 0.5.



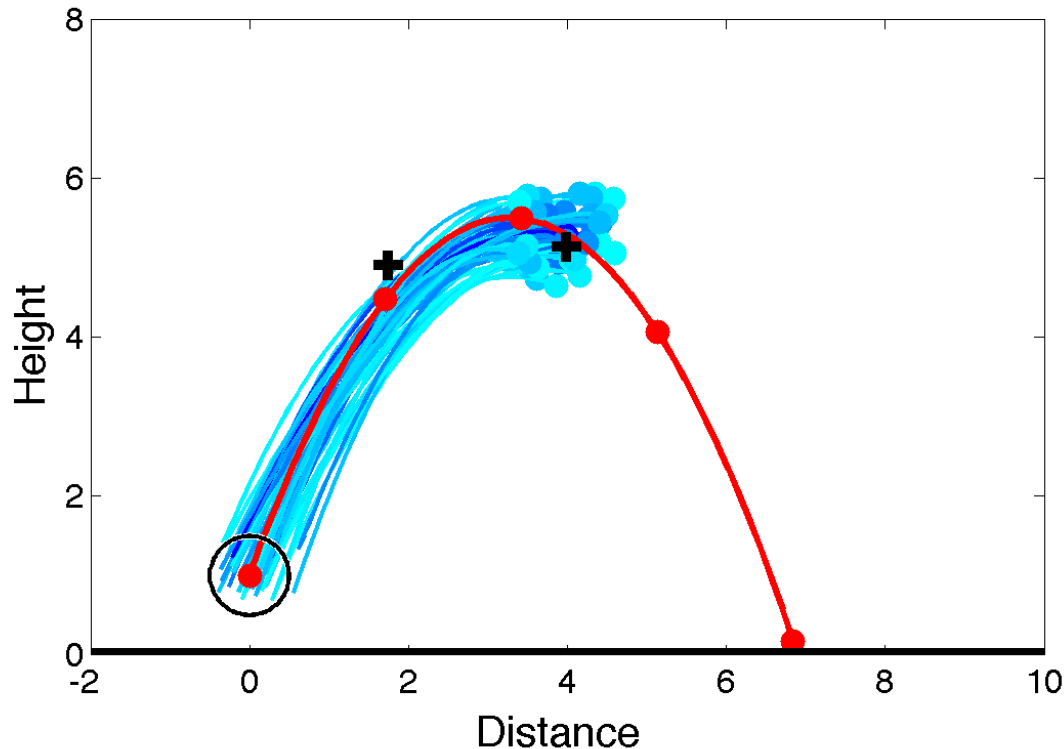
Building a Forecast System



Assimilating the Second Observation

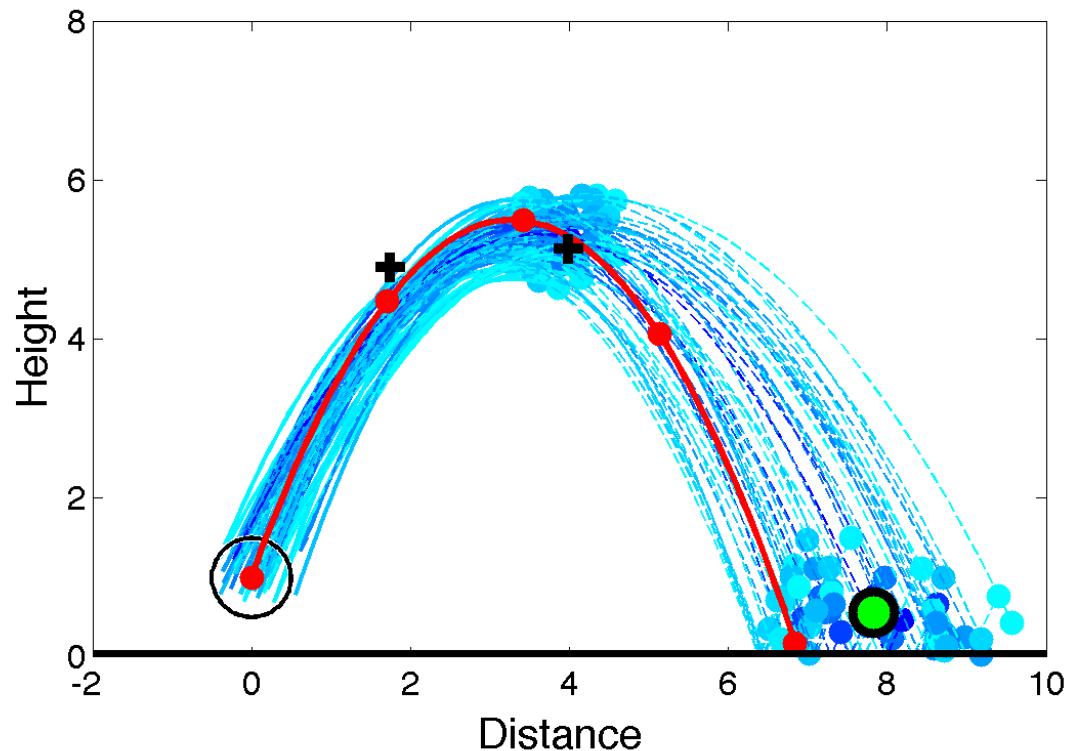
Start with forecast at time 1.0 that used observations at time 0.5.

Add information from observation at time 1.0.



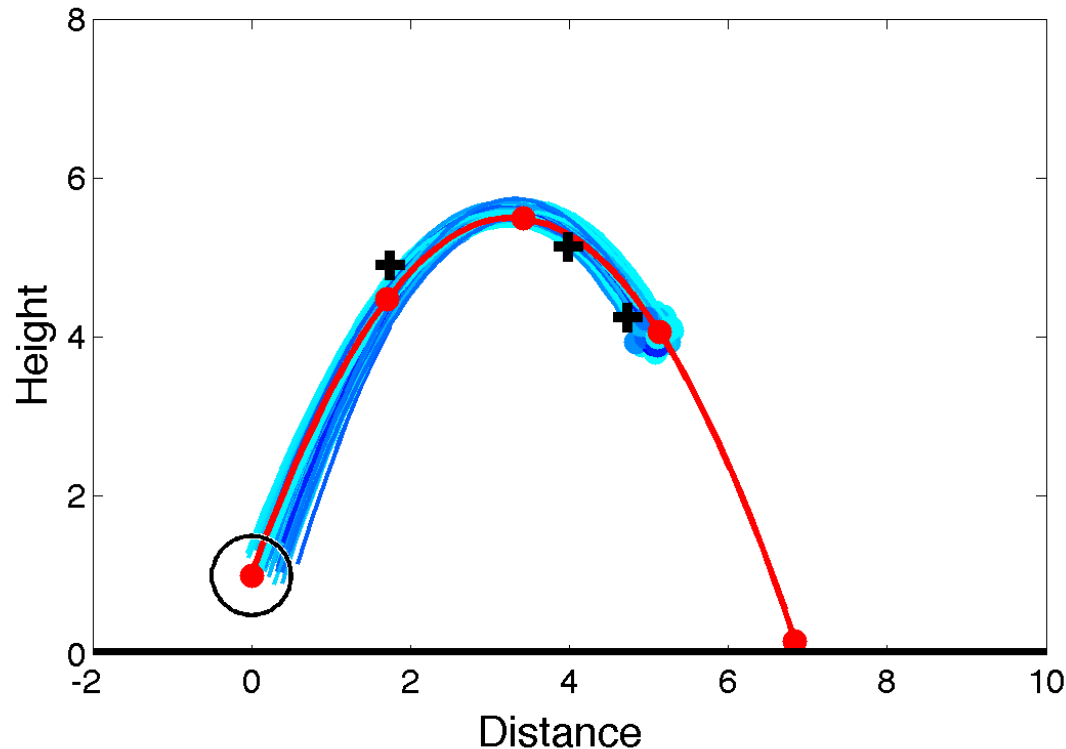
Assimilating the Second Observation

New ensemble analysis is initial conditions for 50 forecasts. Green is best single forecast of red ball at time 2.0 given observations at time 0.5 and 1.0.



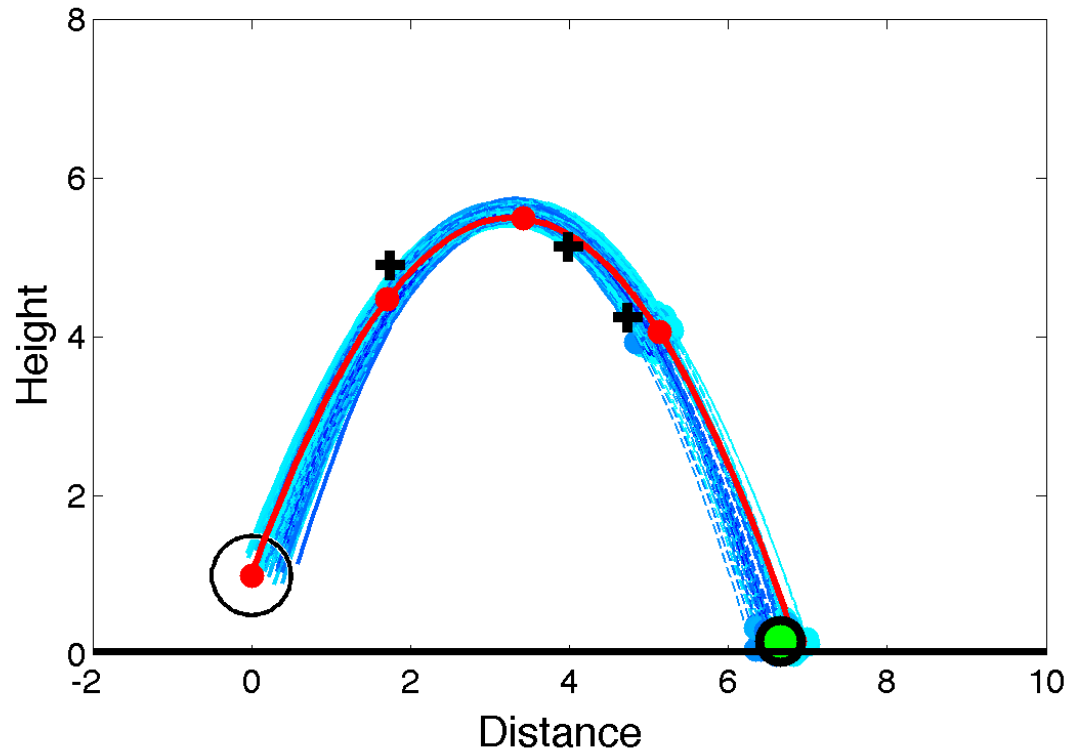
Assimilating the Third Observation

Next ensemble analysis is initial conditions for 50 forecasts.



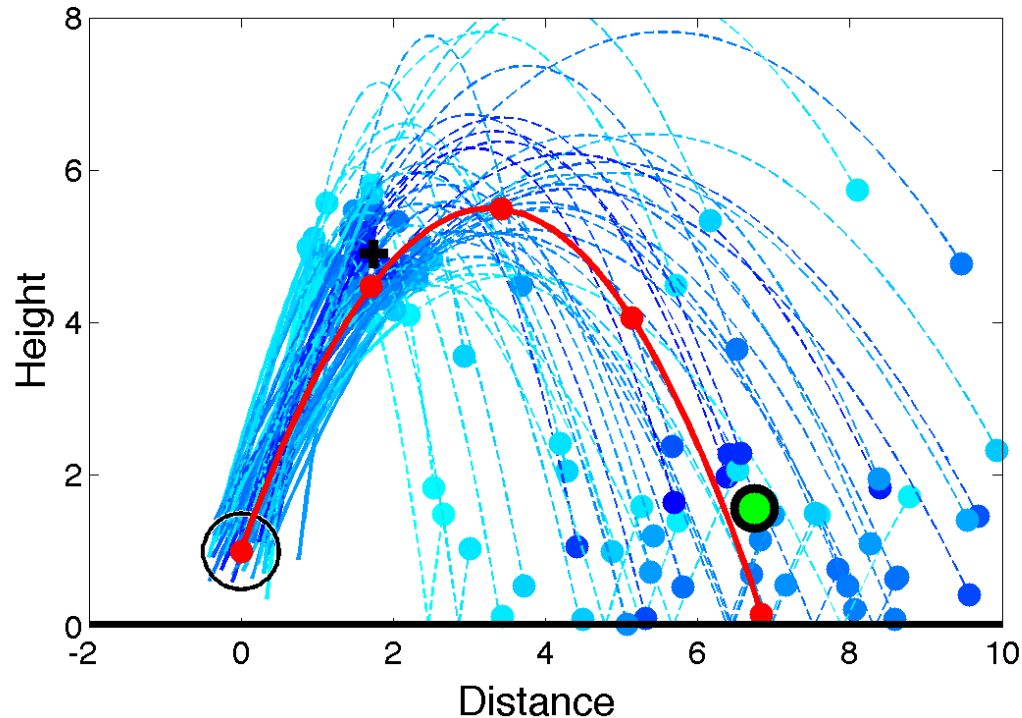
Assimilating the Third Observation

Next ensemble analysis is initial conditions for 50 forecasts. Green is best single forecast of red ball at time 2.0 given observations at time 0.5, 1.0 and 1.5.



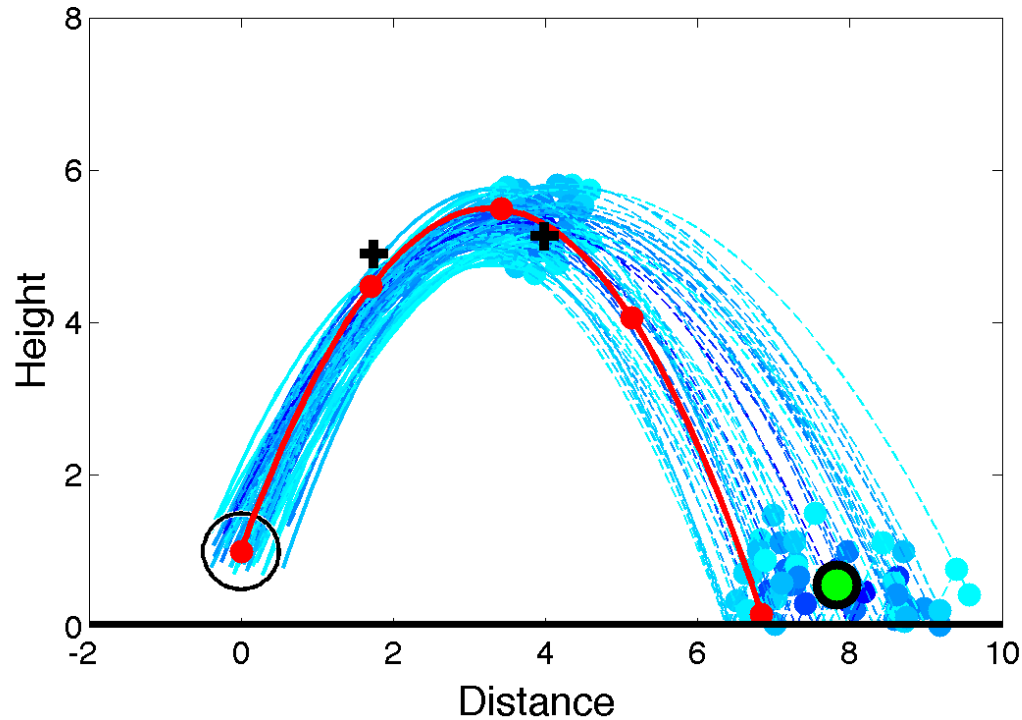
Building a Forecast System

As forecast lead time gets shorter, forecast improves.
Get an estimate of forecast uncertainty.



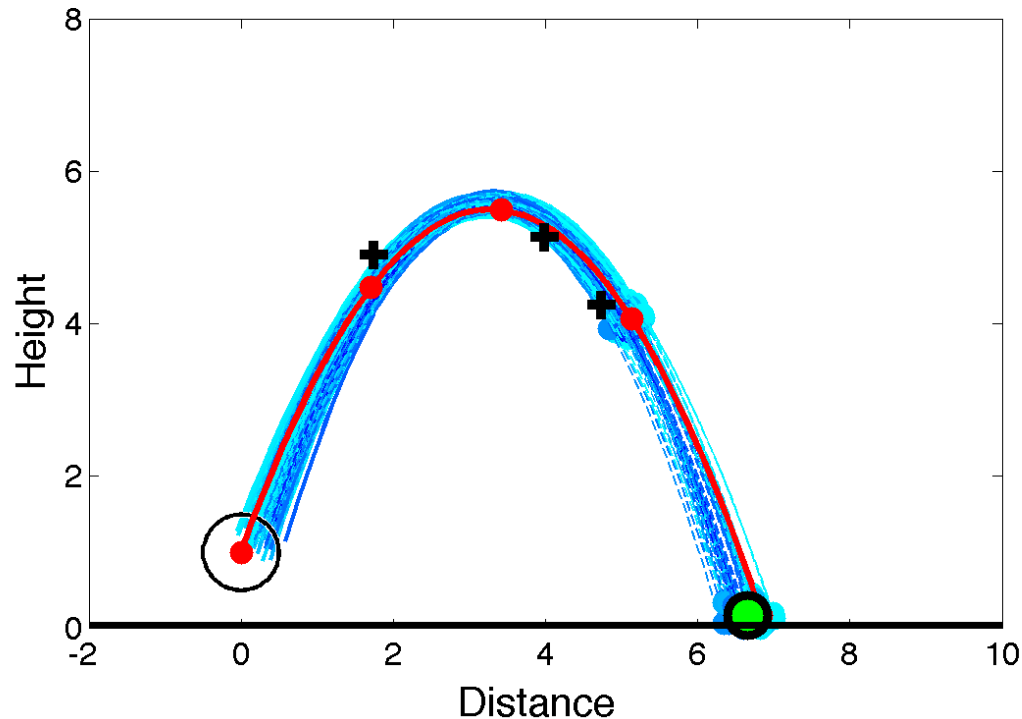
Building a Forecast System

As forecast lead time gets shorter, forecast improves.
Get an estimate of forecast uncertainty.



Building a Forecast System

As forecast lead time gets shorter, forecast improves.
Get an estimate of forecast uncertainty.

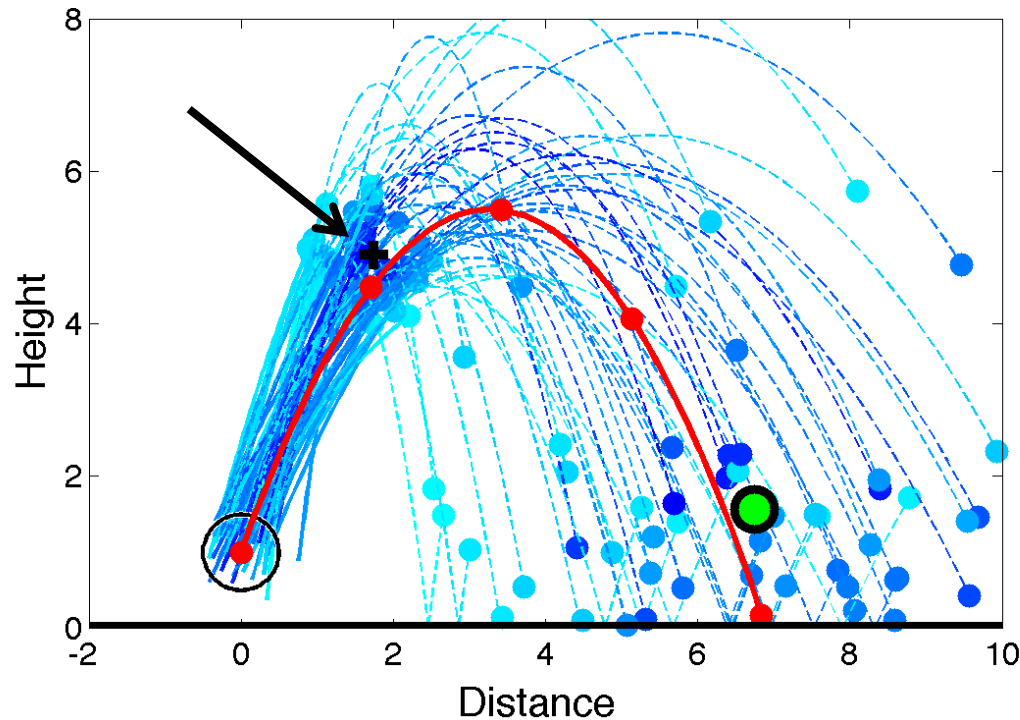


Building a Forecast System

Also can get 'reanalysis' for time 0.5.

Reanalysis uses past and future observations to get best possible estimate of where the red ball was.

Cannot be done in real time.

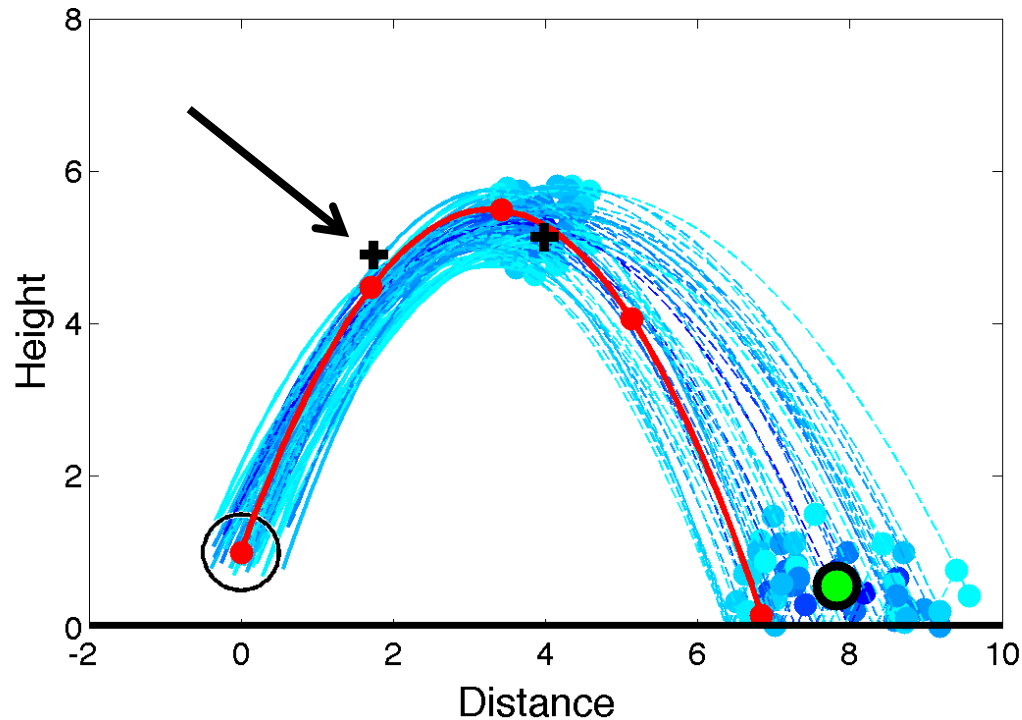


Building a Forecast System

Also can get 'reanalysis' for time 0.5.

Reanalysis uses past and future observations to get best possible estimate of where the red ball was.

Cannot be done in real time.

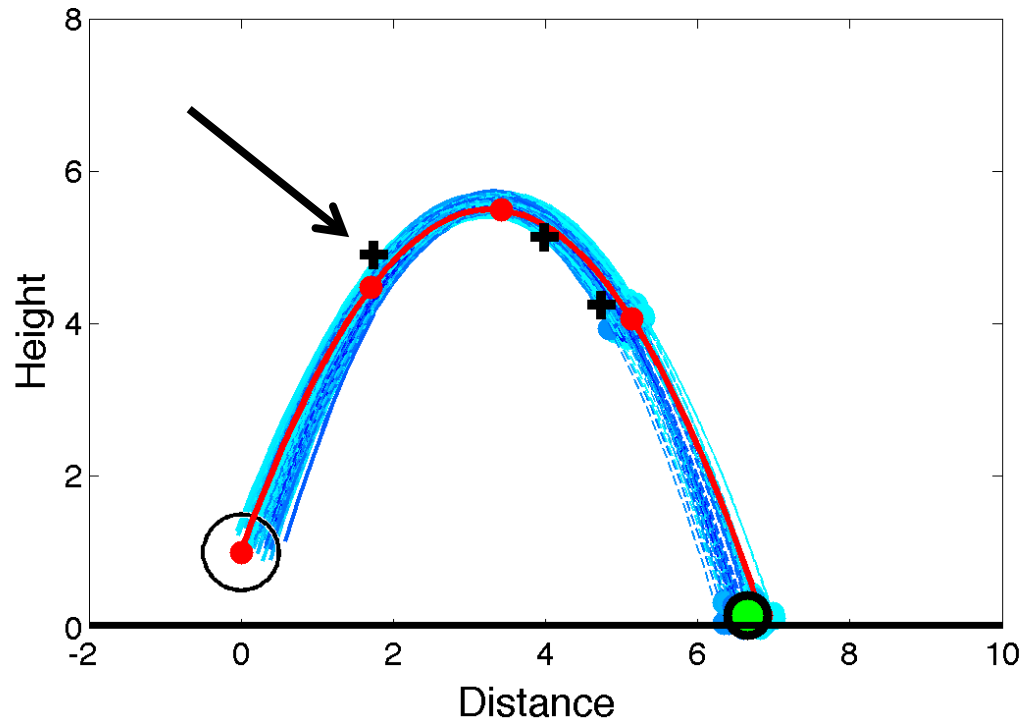


Building a Forecast System

Also can get 'reanalysis' for time 0.5.

Reanalysis uses past and future observations to get best possible estimate of where the red ball was.

Cannot be done in real time.



Building a Forecast System

This thrown ball example is in a 2-dimensional space.
Really a 4-dimensional 'phase' space including velocity.

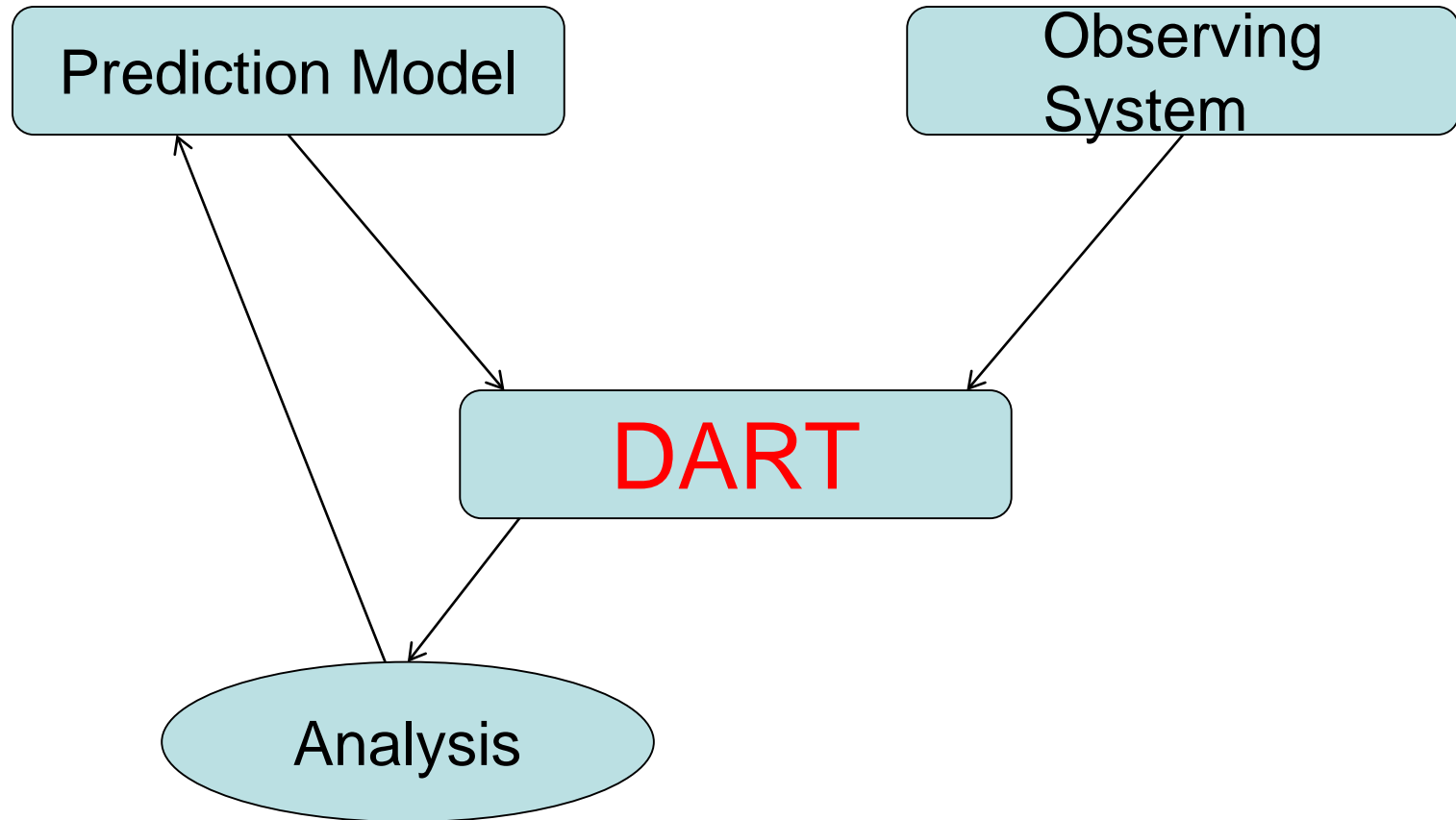
Atmosphere, ocean, land, coupled models are BIG.

But they're still just a 'ball' moving in a HUGE phase space.

As many as 100 million dimensions at present.

An Ensemble Kalman Filter DA Facility (DART)

DART provides data assimilation 'glue' to build state-of-the-art ensemble forecast systems for even the largest models.



DART Design Goals

Provide State-of-the-Art Data Assimilation capability to:

- Prediction research scientists,
- Model developers,
- Observation system developers,

Who may not have any assimilation expertise.

DART Design Constraints

- Models small to huge.
- Few or many observations.
- Tiny to huge computational resources.
- Entry cost must be low.
- Competitive with existing methods for weather prediction:
 - Scientific quality of results,
 - Total computational effort.

A General Description of the Forecast Problem

A system governed by (stochastic) Difference Equation:

$$dx_t = f(x_t, t) + G(x_t, t) d\beta_t, \quad t \geq 0 \quad (1)$$

Observations at discrete times:

$$y_k = h(x_k, t_k) + v_k; \quad k = 1, 2, \dots; \quad t_{k+1} > t_k \geq t_0 \quad (2)$$

Observational error white in time and Gaussian (nice, not essential).

$$v_k \rightarrow N(0, R_k) \quad (3)$$

Complete history of observations is:

$$Y_\tau = \{y_l; t_l \leq \tau\} \quad (4)$$

Goal: Find probability distribution for state:

$$p(x, t | Y_t) \quad \text{Analysis} \quad p(x, t^+ | Y_t) \quad \text{Forecast} \quad (5)$$

A General Description of the Forecast Problem

State between observation times obtained from Difference Equation.
Need to update state given new observations:

$$p(x, t_k | Y_{t_k}) = p(x, t_k | y_k, Y_{t_{k-1}}) \quad (6)$$

Apply Bayes' rule:

$$p(x, t_k | Y_{t_k}) = \frac{p(y_k | x_k, Y_{t_{k-1}}) p(x, t_k | Y_{t_{k-1}})}{p(y_k | Y_{t_{k-1}})} \quad (7)$$

Noise is white in time (3), so:

$$p(y_k | x_k, Y_{t_{k-1}}) = p(y_k | x_k) \quad (8)$$

Integrate numerator to get normalizing denominator:

$$p(y_k | Y_{t_{k-1}}) = \int p(y_k | x) p(x, t_k | Y_{t_{k-1}}) dx \quad (9)$$

A General Description of the Forecast Problem

Probability after new observation:

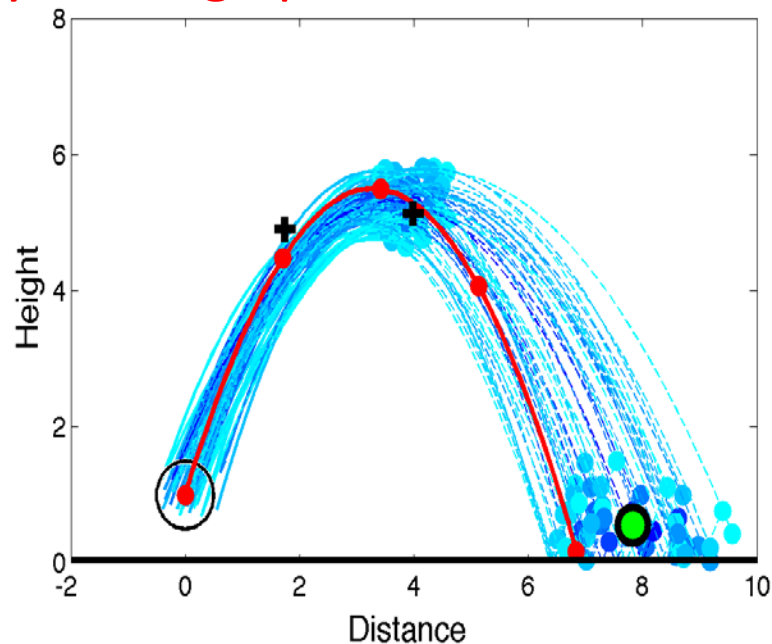
The diagram illustrates the Bayesian update equation (10) with the following components and labels:

- Likelihood:** Points to the term $p(y_k | x)$ in the numerator.
- Prior (forecast):** Points to the term $p(x, t_k | Y_{t_{k-1}})$ in the numerator.
- Posterior (analysis):** Points to the entire left-hand side of the equation, $p(x, t_k | Y_{t_k})$.
- Denominator just normalization:** Points to the integral term in the denominator, $\int p(y_k | \xi) p(\xi, t_k | Y_{t_{k-1}}) d\xi$.

$$p(x, t_k | Y_{t_k}) = \frac{p(y_k | x) p(x, t_k | Y_{t_{k-1}})}{\int p(y_k | \xi) p(\xi, t_k | Y_{t_{k-1}}) d\xi} \quad (10)$$

Methods for Solving the Forecast Problem: Particle Filter

Independent evolving estimates,
Associate probability with each estimate given observations,
Eliminate unlikely estimates,
Duplicate likely estimates,
Can represent arbitrary probability distribution,
Scales very poorly for large problems.



Methods for Solving the Forecast Problem: Variational

Four-Dimensional Variational Method:

- Minimize a cost function motivated by Eq. 10,
- Find optimal fit of evolving model to observations,
- Use variational calculus (adjoint) to compute gradient,
- State-of-the-art for weather prediction until recently.

- Creating model adjoints may require significant effort.
- Only provides estimate of mean state.

Methods for Solving the Forecast Problem: Kalman Filter

Assumes:

linear model

Gaussian noise

$$dx_t = f(x_t, t) + G(x_t, t) d\beta_t, \quad t \geq 0$$

Gaussian state

linear forward operator,

$$y_k = h(x_k, t_k) + v_k; \quad k = 1, 2, \dots; \quad t_{k+1} > t_k \geq t_0$$

Gaussian observation error

Product of Two Gaussians

Product of d-dimensional normals with means μ_1 and μ_2 and covariance matrices Σ_1 and Σ_2 is normal.

$$N(\mu_1, \Sigma_1)N(\mu_2, \Sigma_2) = cN(\mu, \Sigma)$$

Product of Two Gaussians

Product of d-dimensional normals with means μ_1 and μ_2 and covariance matrices Σ_1 and Σ_2 is normal.

$$N(\mu_1, \Sigma_1)N(\mu_2, \Sigma_2) = cN(\mu, \Sigma)$$

Covariance: $\Sigma = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$

Mean: $u = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}(\Sigma_1^{-1} \mu_1 + \Sigma_2^{-1} \mu_2)$

Product of Two Gaussians

Product of d-dimensional normals with means μ_1 and μ_2 and covariance matrices Σ_1 and Σ_2 is normal.

$$N(\mu_1, \Sigma_1)N(\mu_2, \Sigma_2) = cN(\mu, \Sigma)$$

Covariance: $\Sigma = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$

Mean: $\mu = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}(\Sigma_1^{-1} \mu_1 + \Sigma_2^{-1} \mu_2)$

Weight: $c = \frac{1}{(2\Pi)^{d/2} |\Sigma_1 + \Sigma_2|^{1/2}} \exp \left\{ -\frac{1}{2} [(\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1)] \right\}$

We'll ignore the weight since we immediately normalize products to be PDFs.

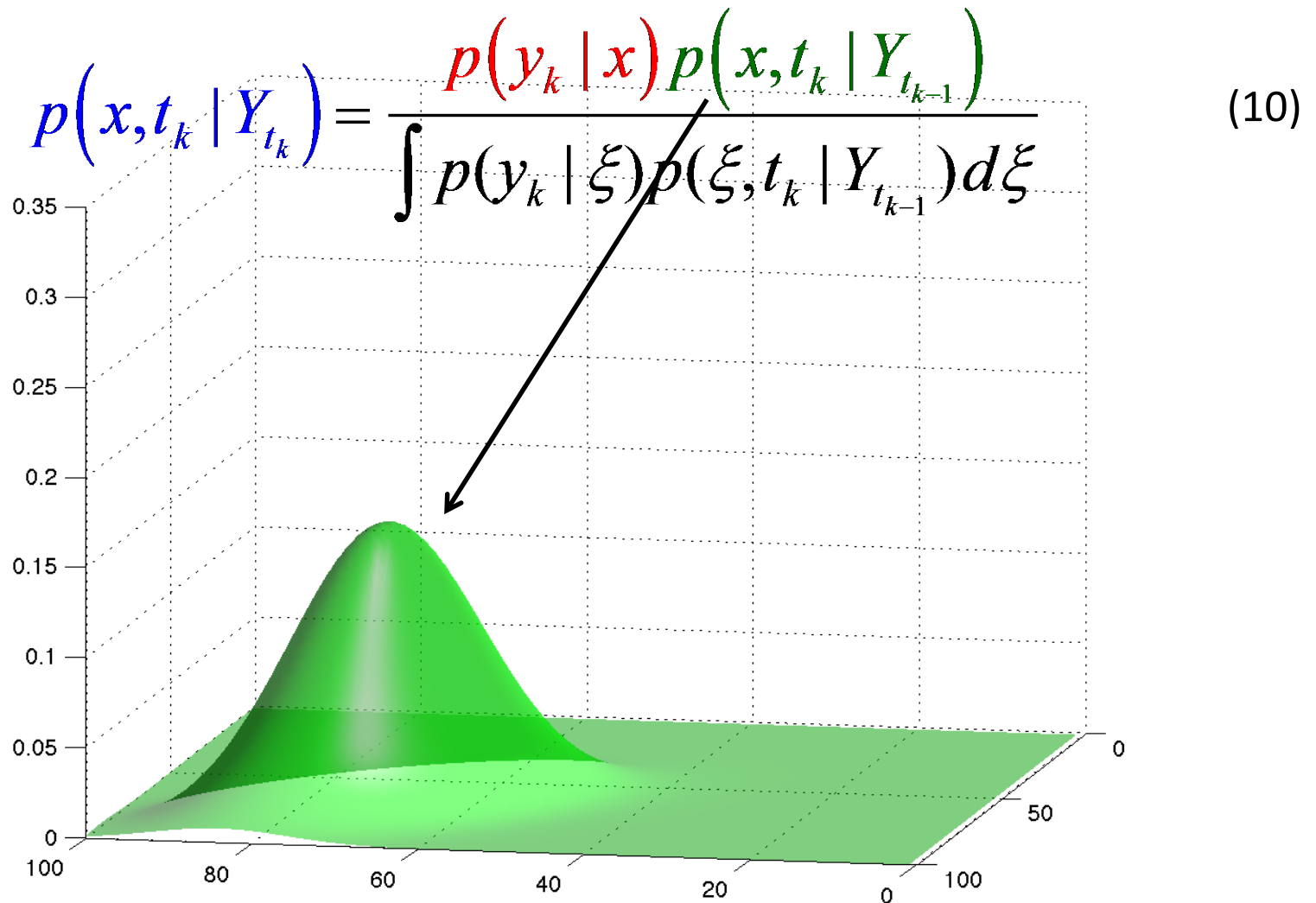
The Kalman Filter

$$p(x, t_k | Y_{t_k}) = \frac{p(y_k | x) p(x, t_k | Y_{t_{k-1}})}{\int p(y_k | \xi) p(\xi, t_k | Y_{t_{k-1}}) d\xi} \quad (10)$$

Numerator is just product of two gaussians.

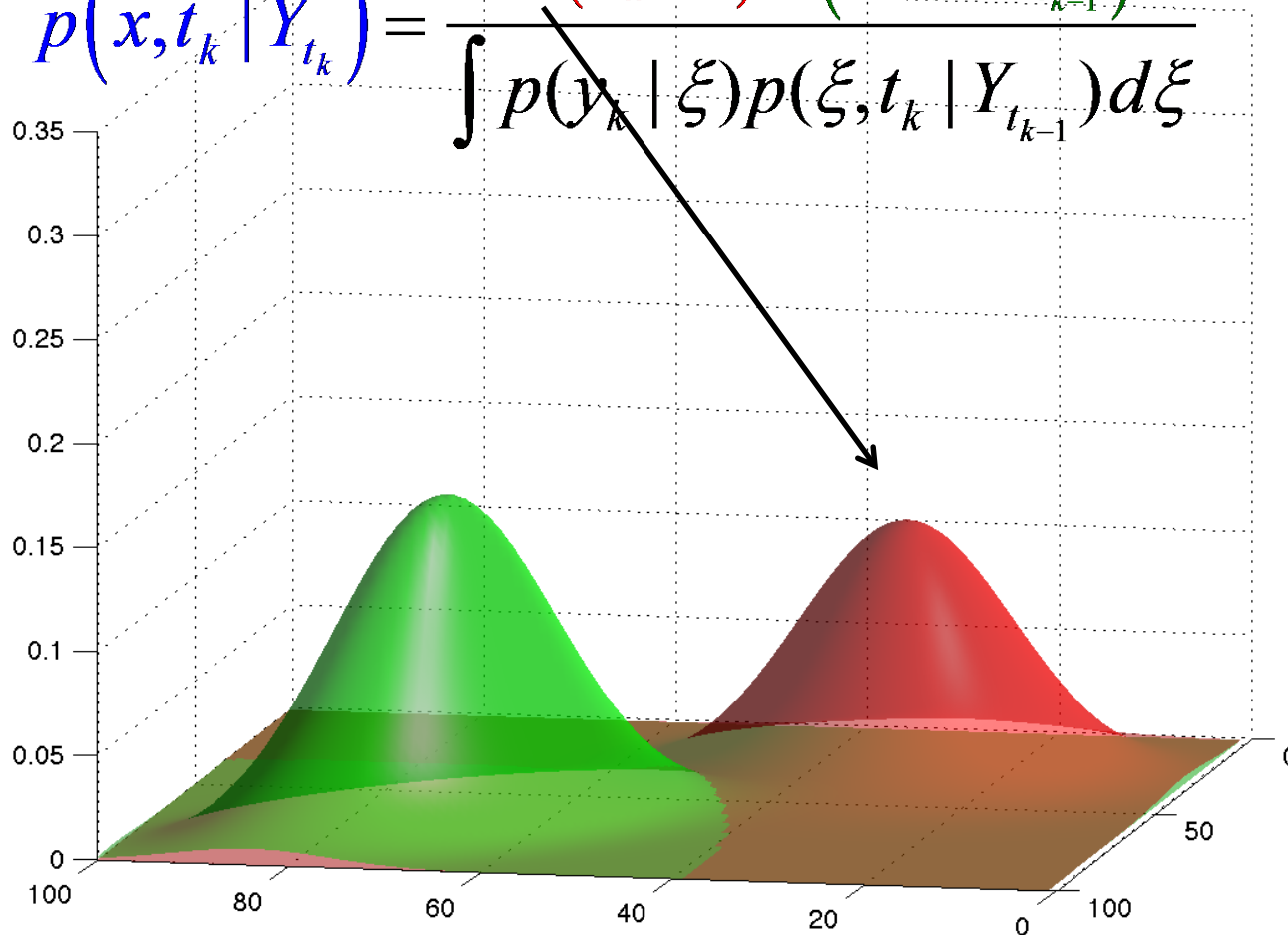
Denominator just normalizes posterior to be a PDF.

The Kalman Filter



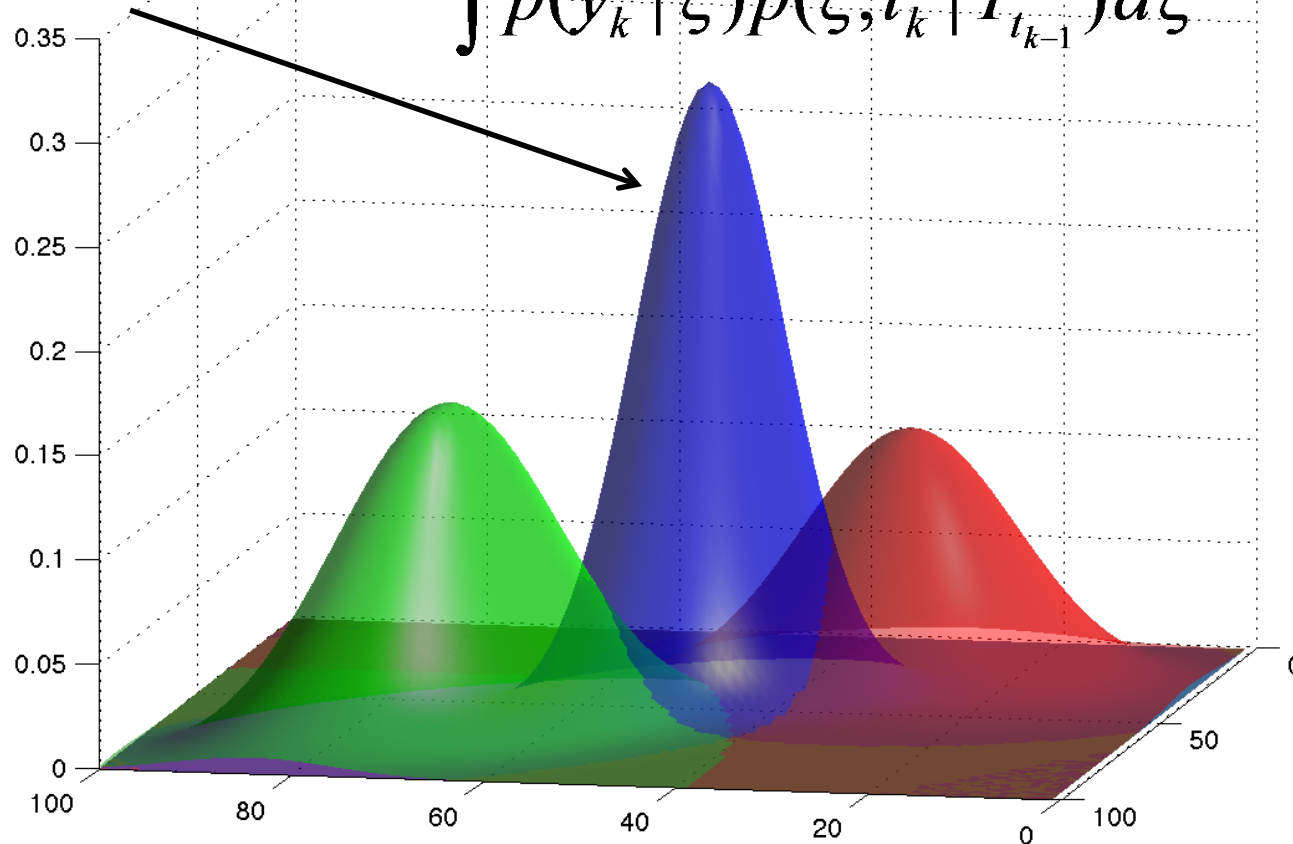
The Kalman Filter

$$p(x, t_k | Y_{t_k}) = \frac{p(y_k | x) p(x, t_k | Y_{t_{k-1}})}{\int p(y_k | \xi) p(\xi, t_k | Y_{t_{k-1}}) d\xi} \quad (10)$$



The Kalman Filter

$$p(x, t_k | Y_{t_k}) = \frac{p(y_k | x) p(x, t_k | Y_{t_{k-1}})}{\int p(y_k | \xi) p(\xi, t_k | Y_{t_{k-1}}) d\xi} \quad (10)$$



Kalman Filter: Cost Challenges

Product of d-dimensional normals with means μ_1 and μ_2 and covariance matrices Σ_1 and Σ_2 is normal.

$$N(\mu_1, \Sigma_1)N(\mu_2, \Sigma_2) = cN(\mu, \Sigma)$$

Covariance: $\Sigma = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$

Mean: $u = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}(\Sigma_1^{-1} \mu_1 + \Sigma_2^{-1} \mu_2)$

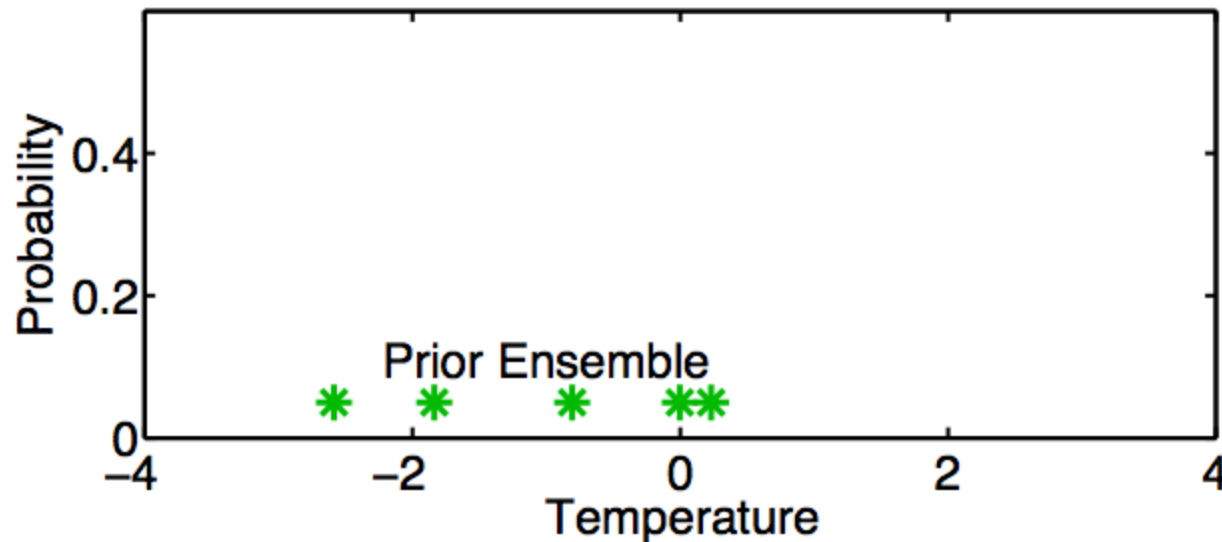
Must store and invert covariance matrices.

Too big to store for large problems.

Too costly to invert, $> O(n^2)$.

A One-Dimensional Ensemble Kalman Filter

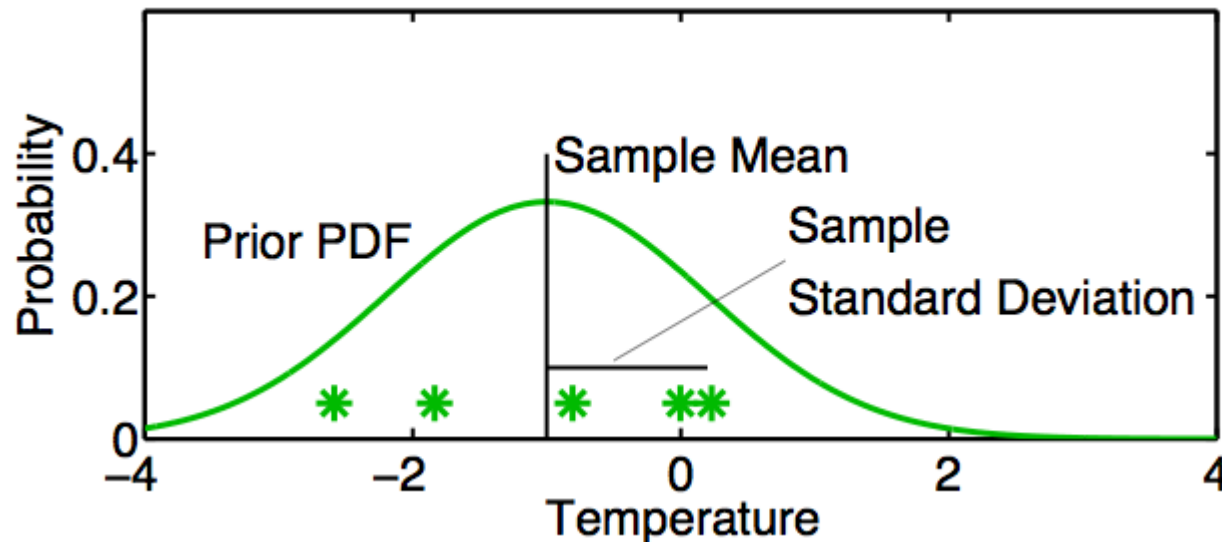
Represent a prior pdf by a sample (ensemble) of N values:



Example: Predict temperature on a cold day at NCAR.

A One-Dimensional Ensemble Kalman Filter

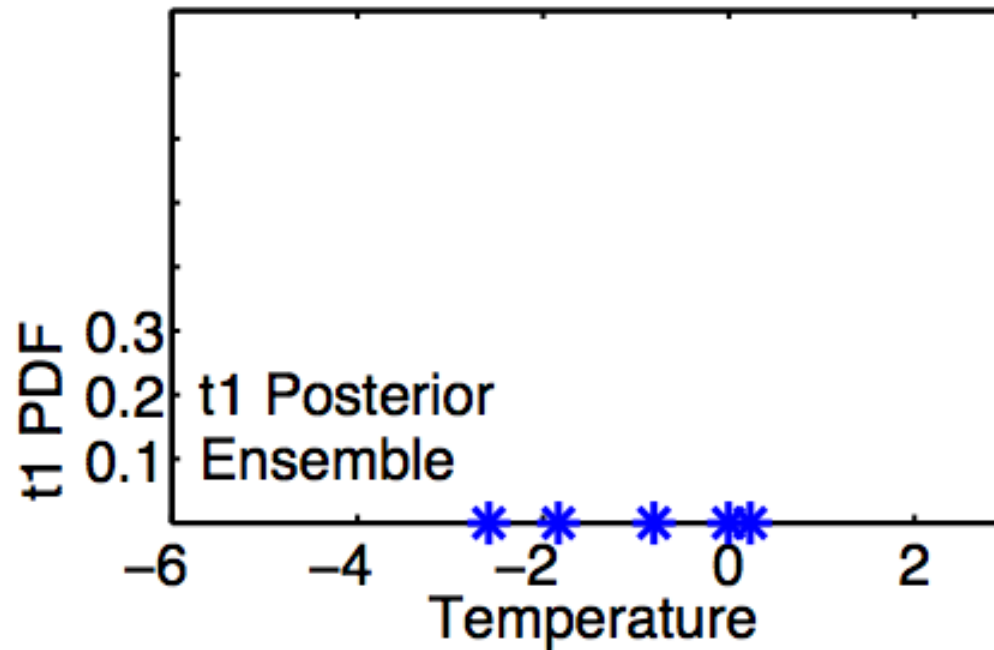
Represent a prior pdf by a sample (ensemble) of N values:



Use sample mean $\bar{T} = \sum_{n=1}^N T_n / N$
and sample standard deviation $\sigma_T = \sqrt{\sum_{n=1}^N (T_n - \bar{T})^2 / (N-1)}$
to determine a corresponding continuous distribution $Normal(\bar{T}, \sigma_T)$

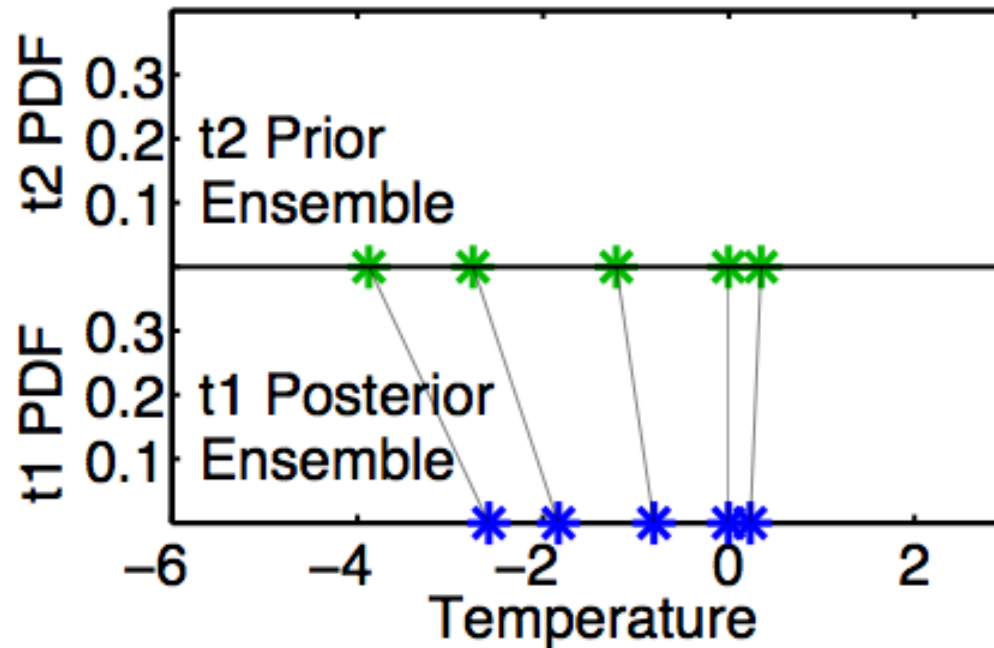
A One-Dimensional Ensemble Kalman Filter: Model Advance

If posterior ensemble at time t_1 is $T_{1,n}$, $n = 1, \dots, N$



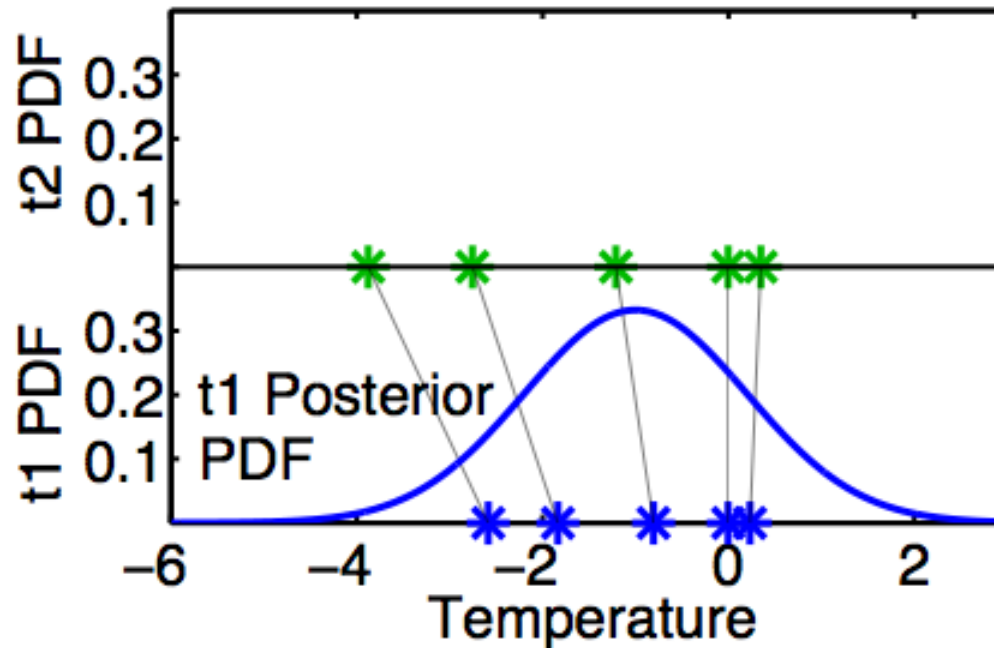
A One-Dimensional Ensemble Kalman Filter: Model Advance

If posterior ensemble at time t_1 is $T_{1,n}$, $n = 1, \dots, N$,
advance each member to time t_2 with model, $T_{2,n} = L(T_{1,n})$ $n = 1, \dots, N$.



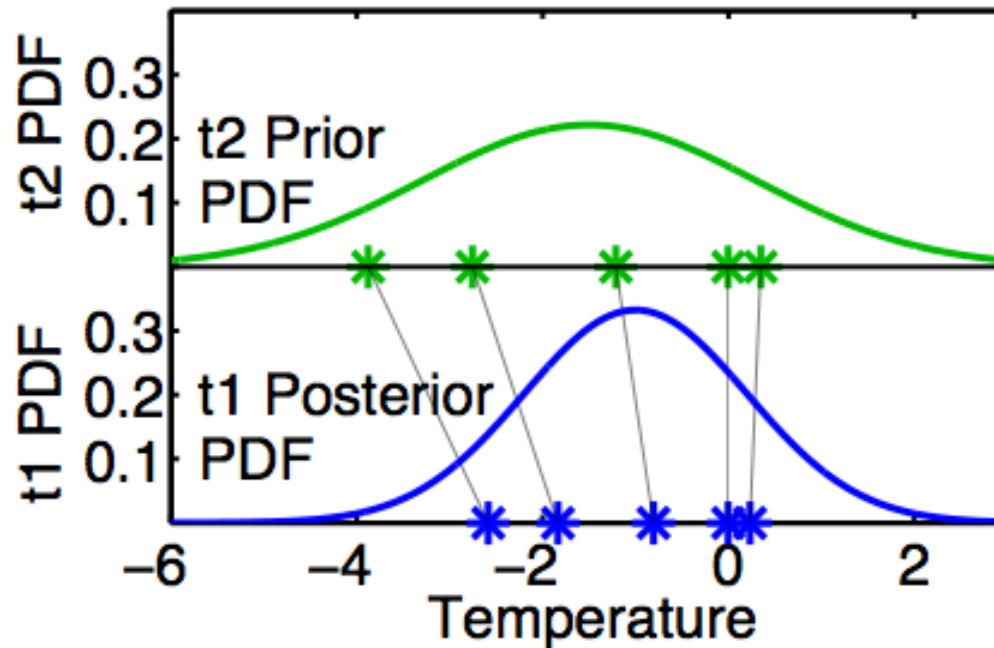
A One-Dimensional Ensemble Kalman Filter: Model Advance

Same as advancing continuous pdf at time t_1 ...

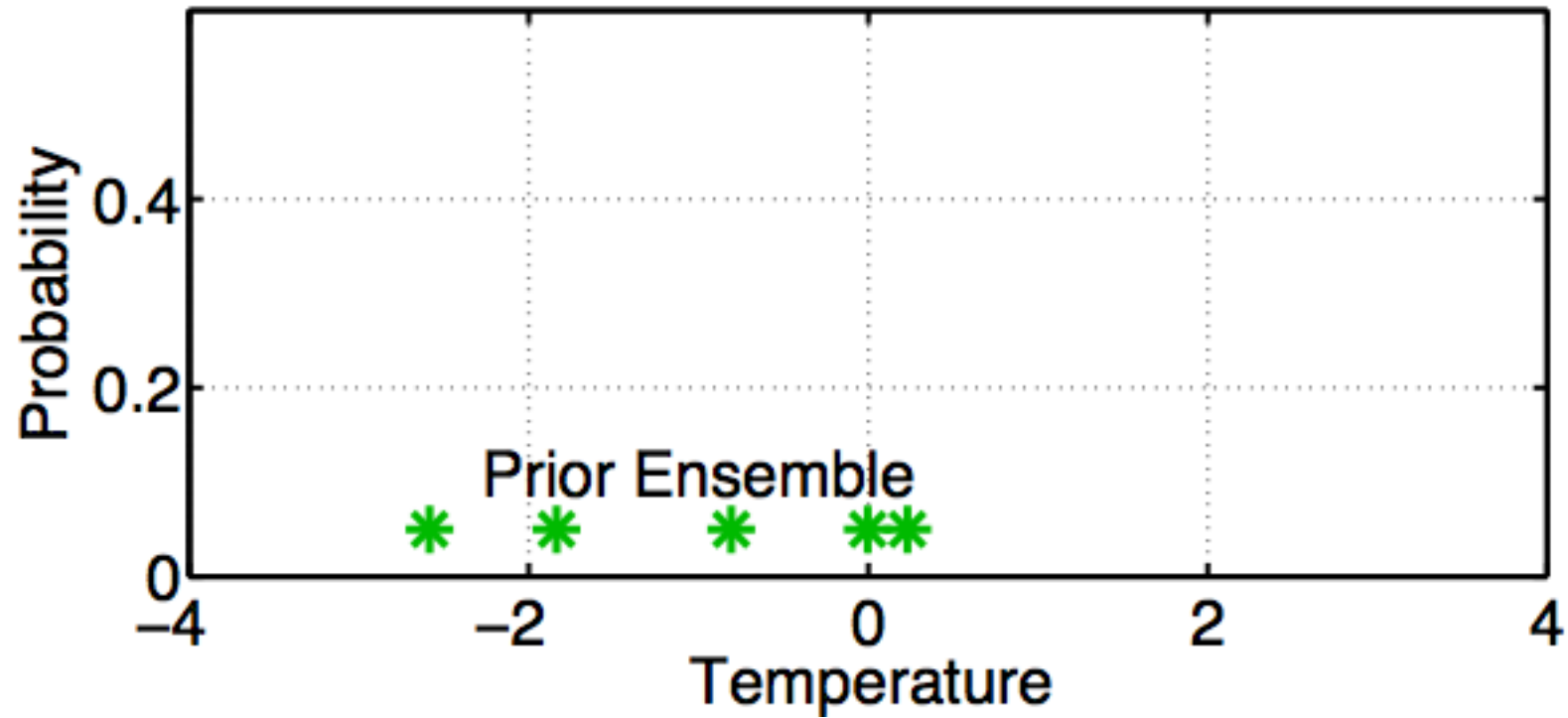


A One-Dimensional Ensemble Kalman Filter: Model Advance

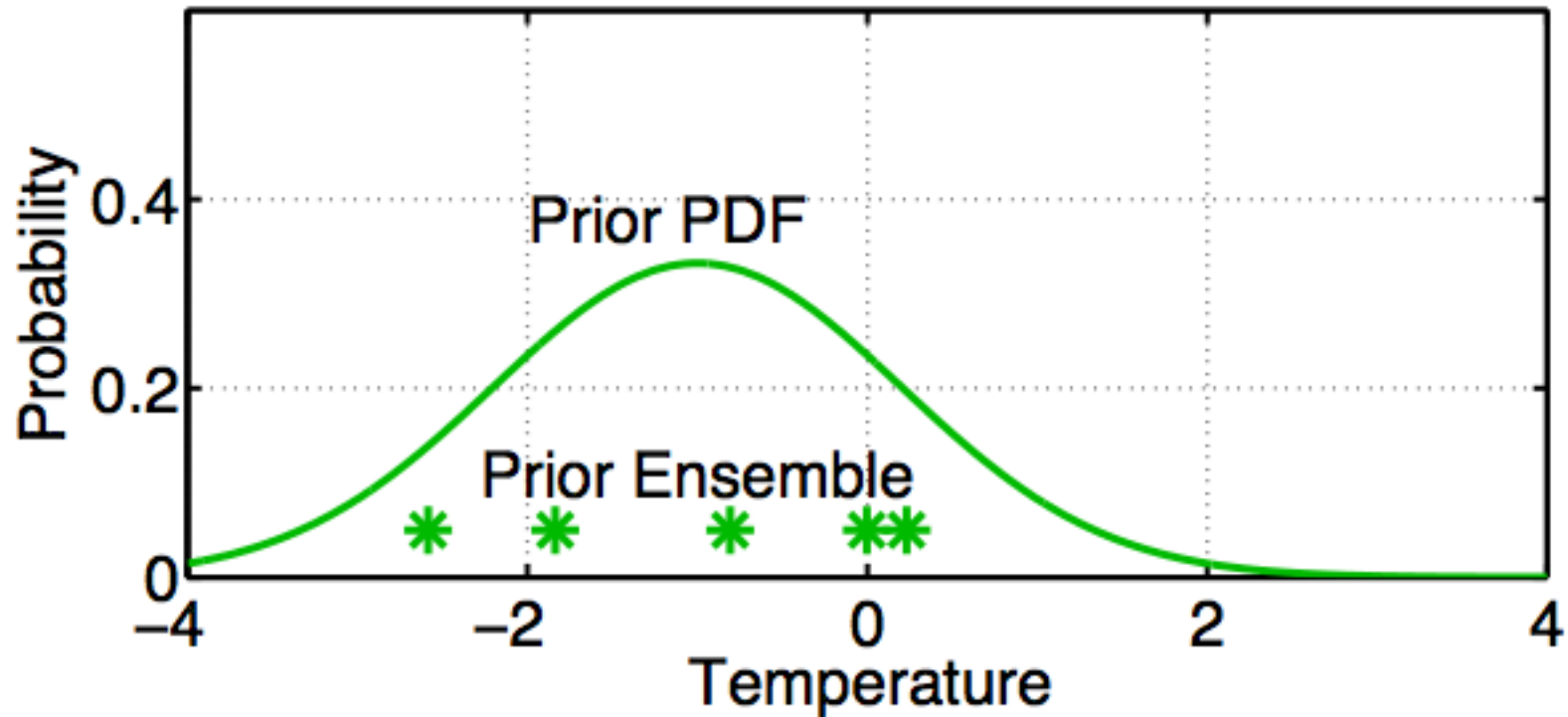
Same as advancing continuous pdf at time t_1 to time t_2 with model L .



One-Dimensional Ensemble Kalman Filter: Assimilating an Observation

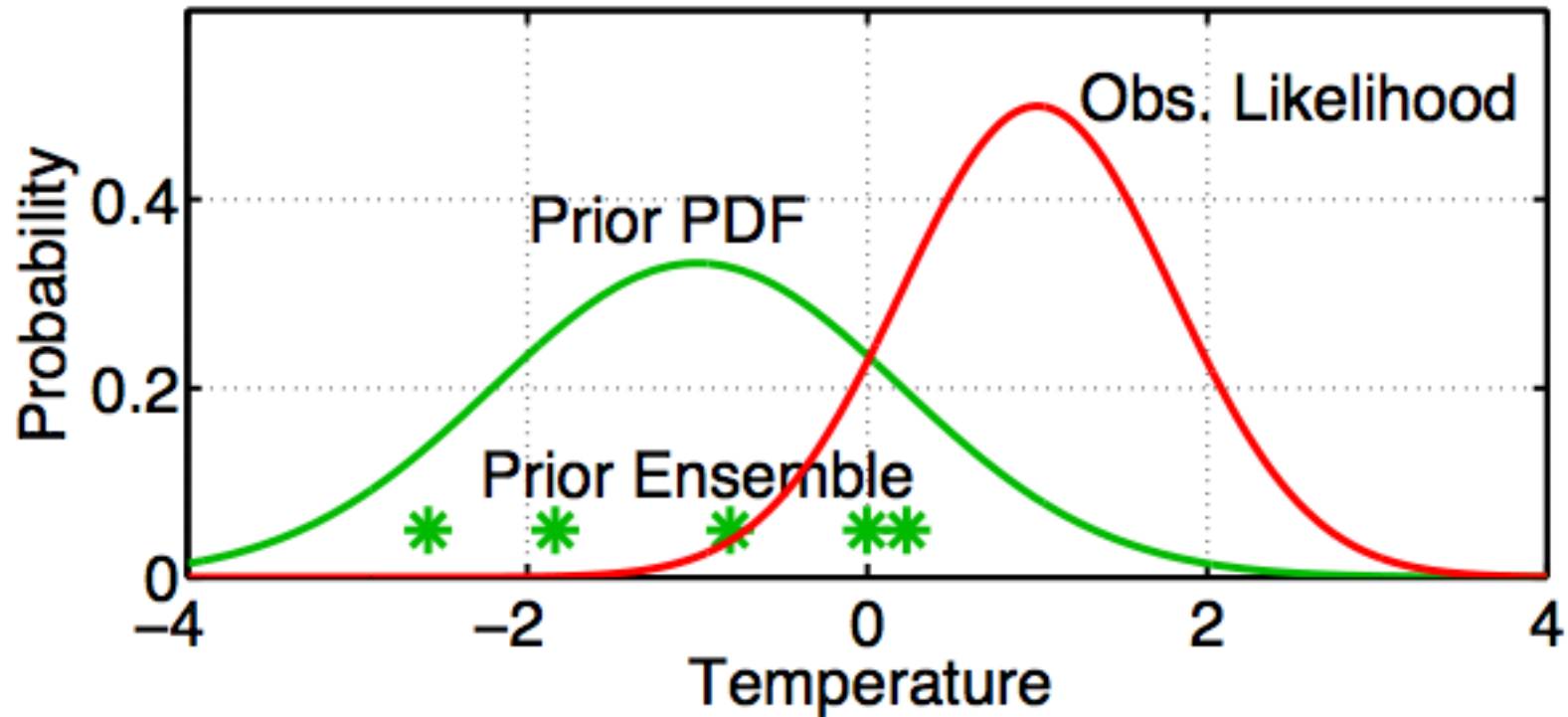


One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



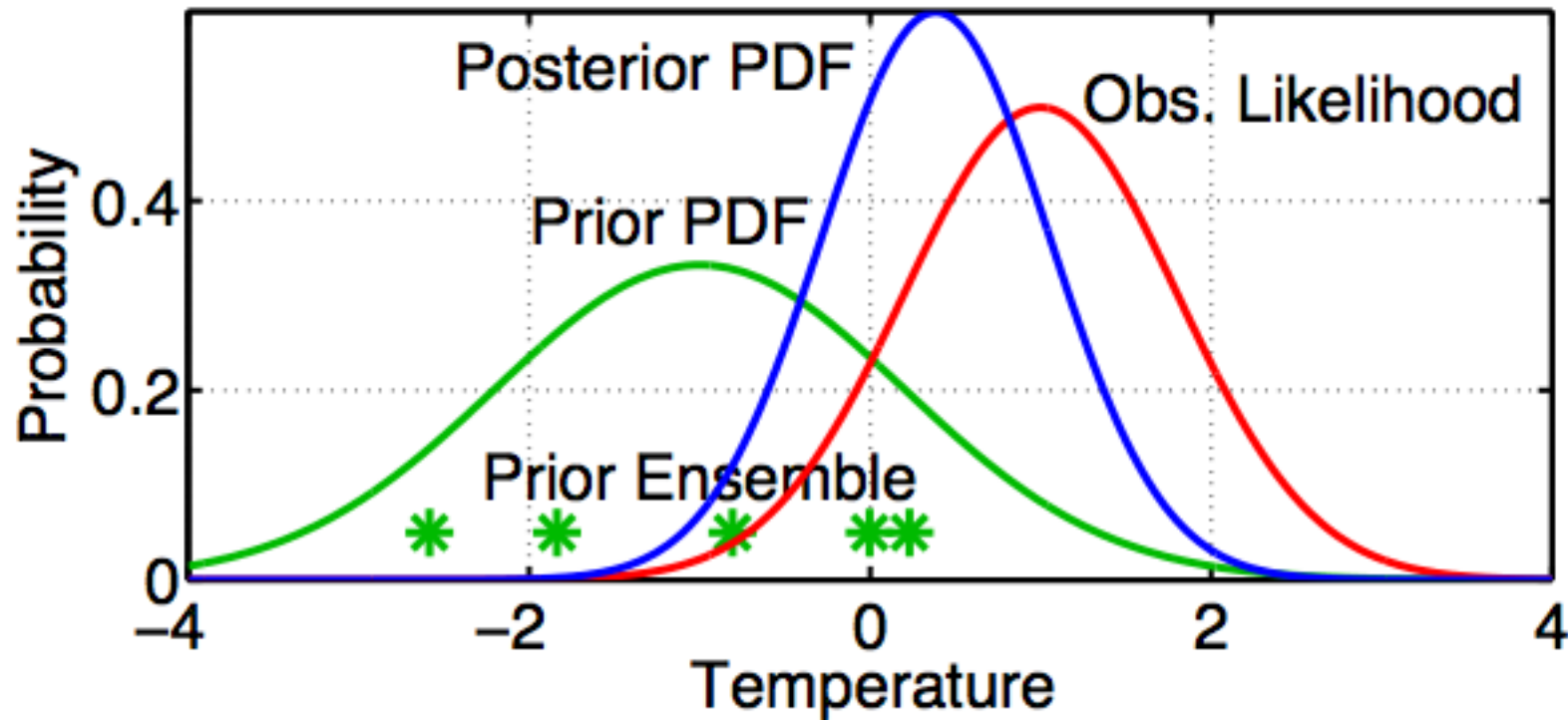
Fit a Gaussian to the sample.

One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



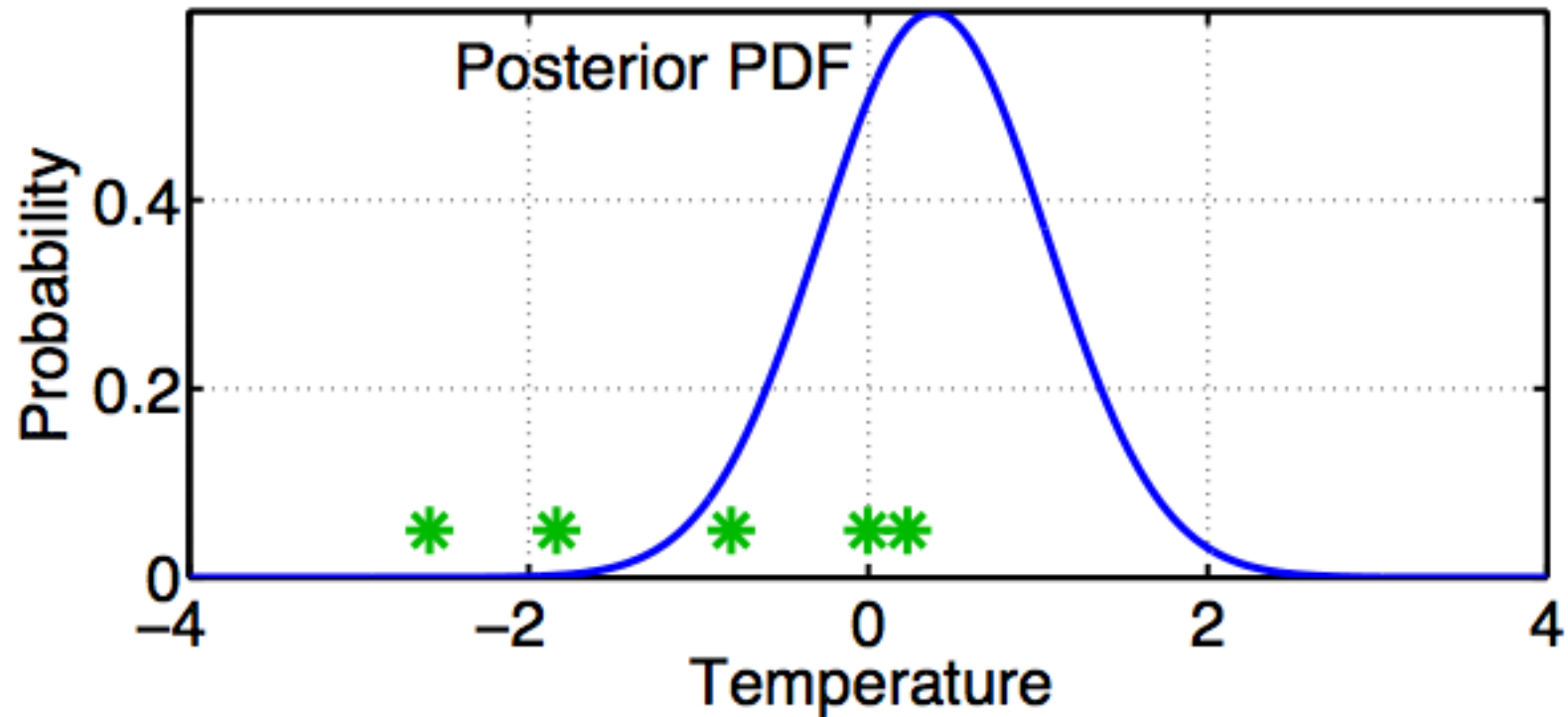
Get the observation likelihood.

One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



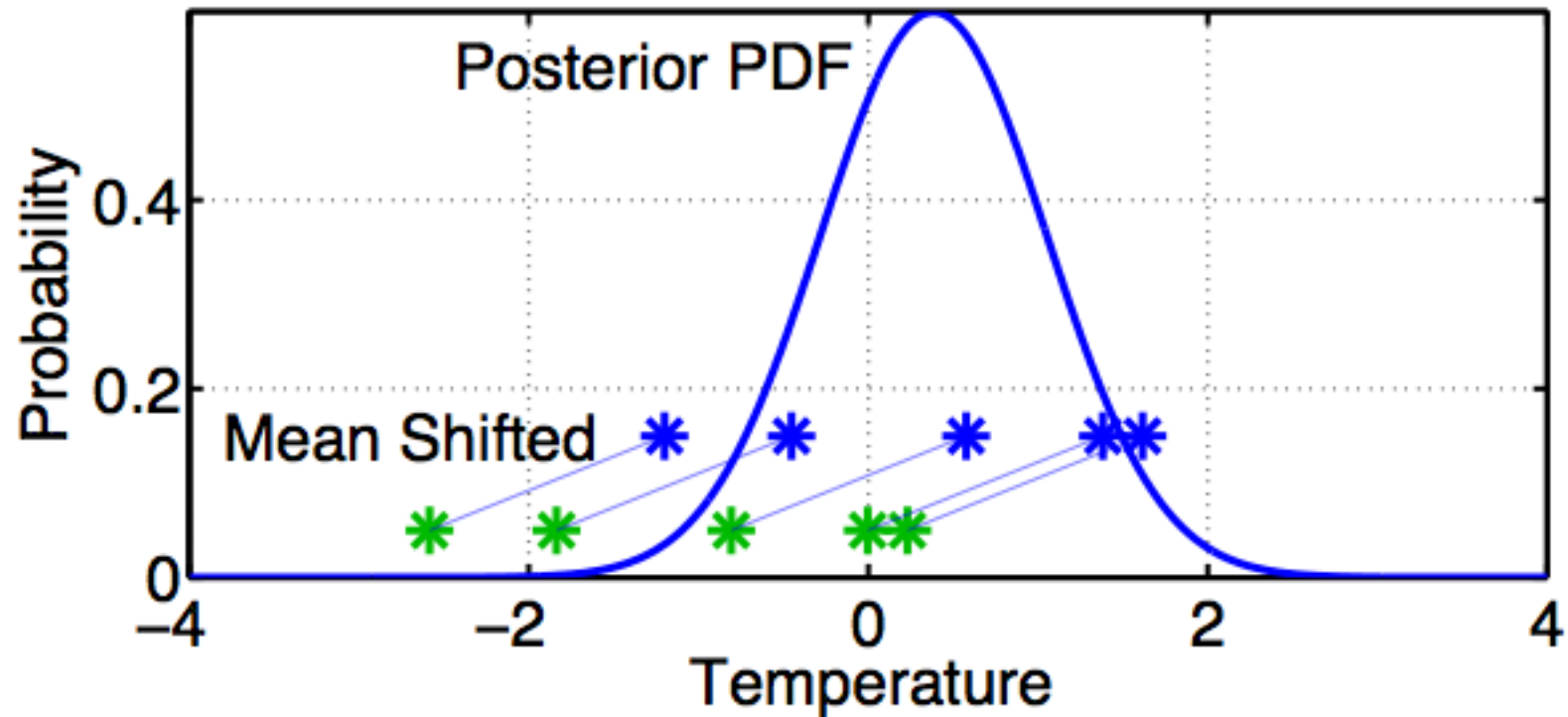
Compute the continuous posterior PDF.

One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



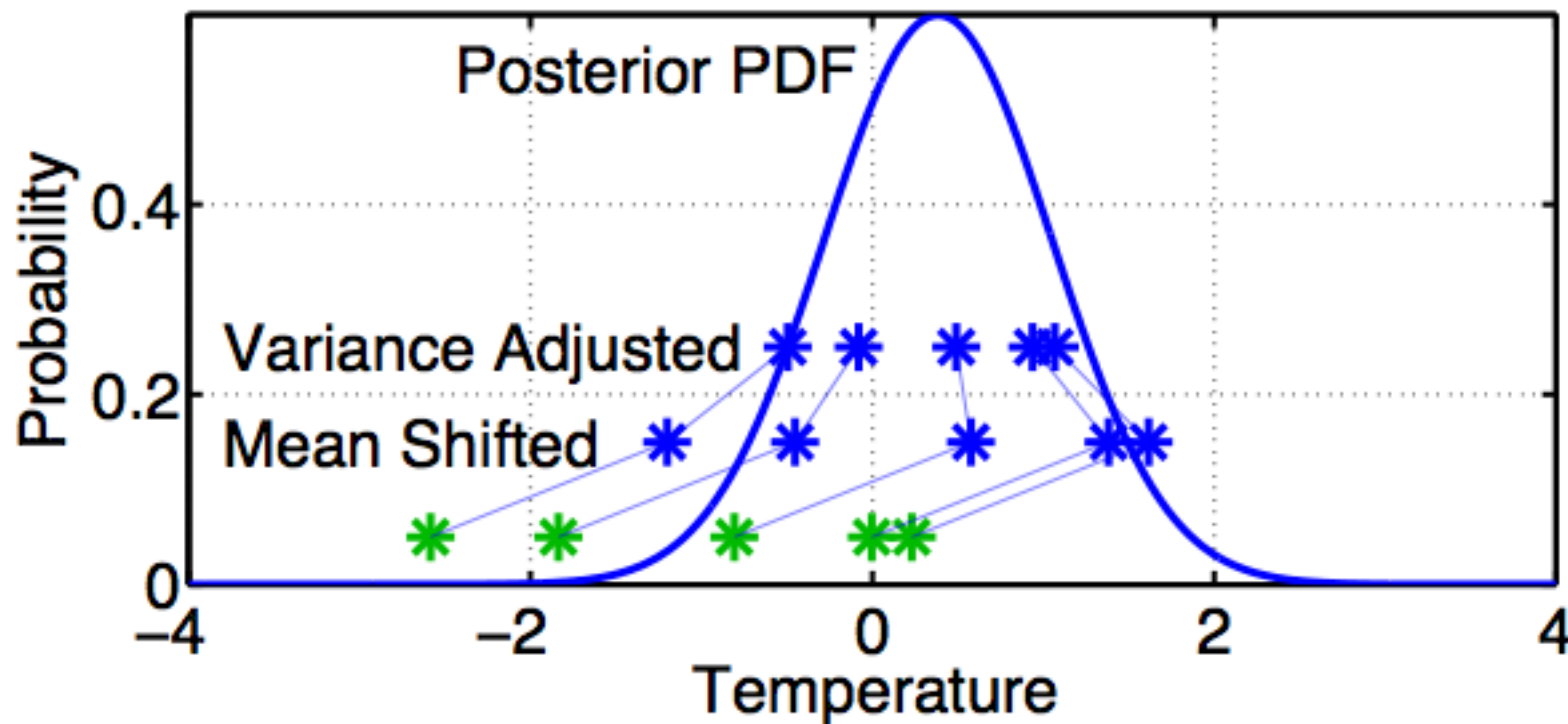
Use a deterministic algorithm to ‘adjust’ the ensemble.

One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



First, ‘shift’ the ensemble to have the exact mean of the posterior.

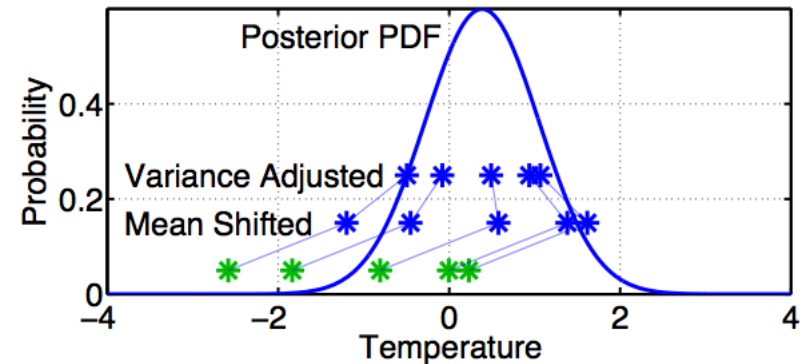
One-Dimensional Ensemble Kalman Filter: Assimilating an Observation



First, ‘shift’ the ensemble to have the exact mean of the posterior.
Second, linearly contract to have the exact variance of the posterior.
Sample statistics are identical to Kalman filter.

Single observed variable, single unobserved variable.

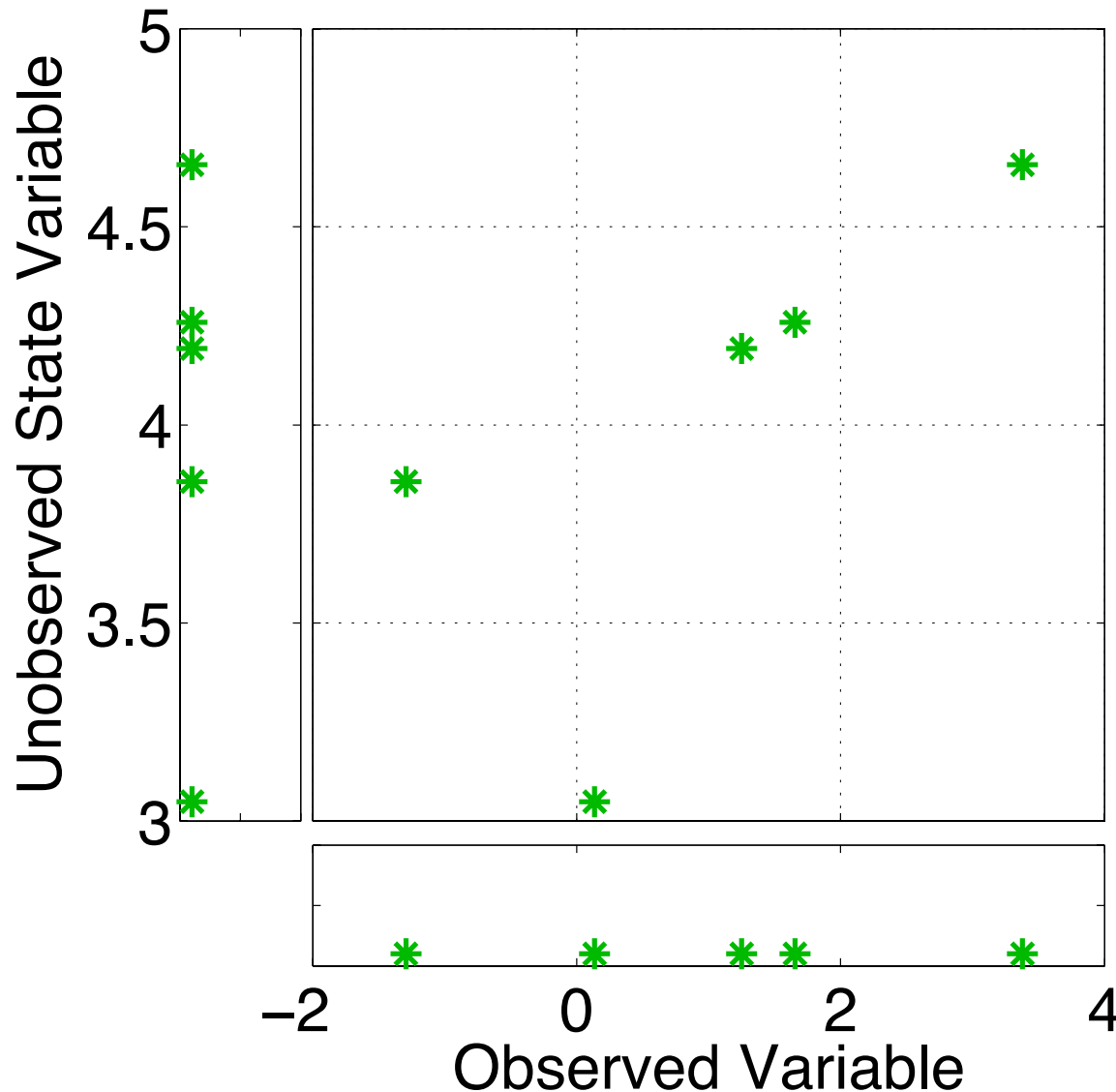
So far, we have a known
likelihood for a



Now, suppose the model state has an additional variable,
temperature at Denver Airport.

How should ensemble members update the additional variable?

Ensemble filters: Updating additional prior state variables

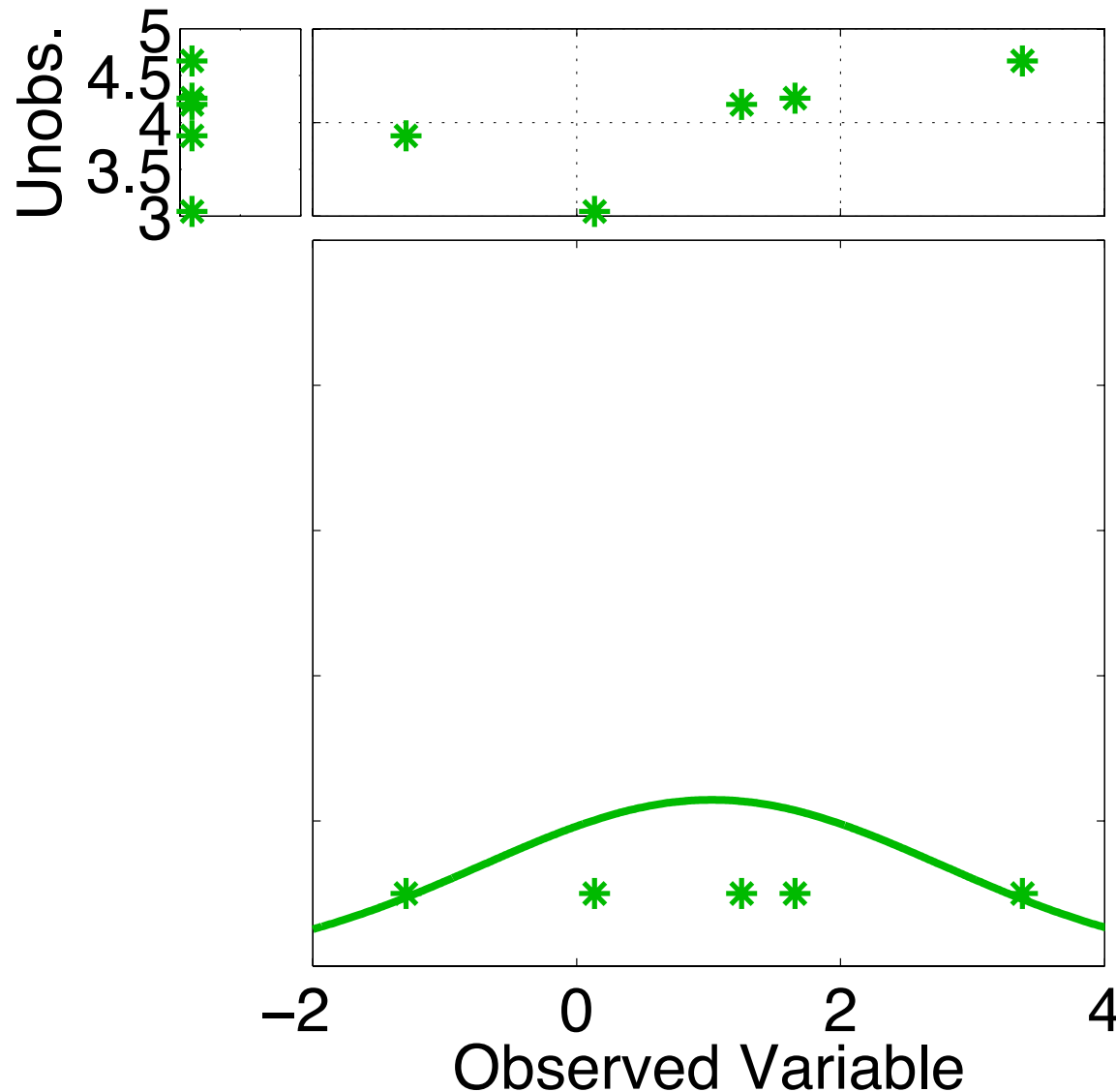


Assume that all we know is the prior joint distribution.

One variable is observed.

What should happen to the unobserved variable?

Ensemble filters: Updating additional prior state variables

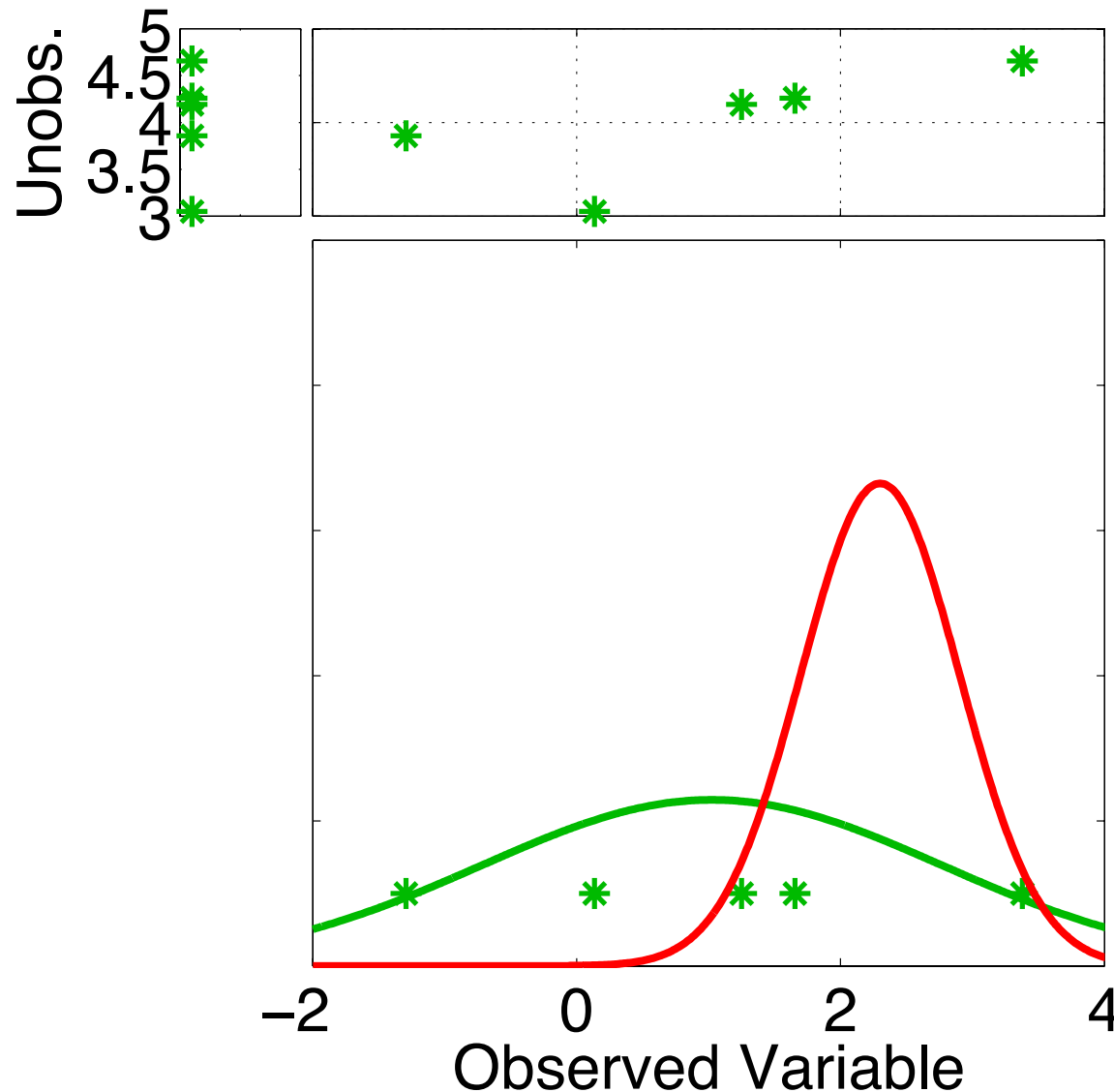


Assume that all we know is the prior joint distribution.

One variable is observed.

Update observed variable with ensemble Kalman filter.

Ensemble filters: Updating additional prior state variables

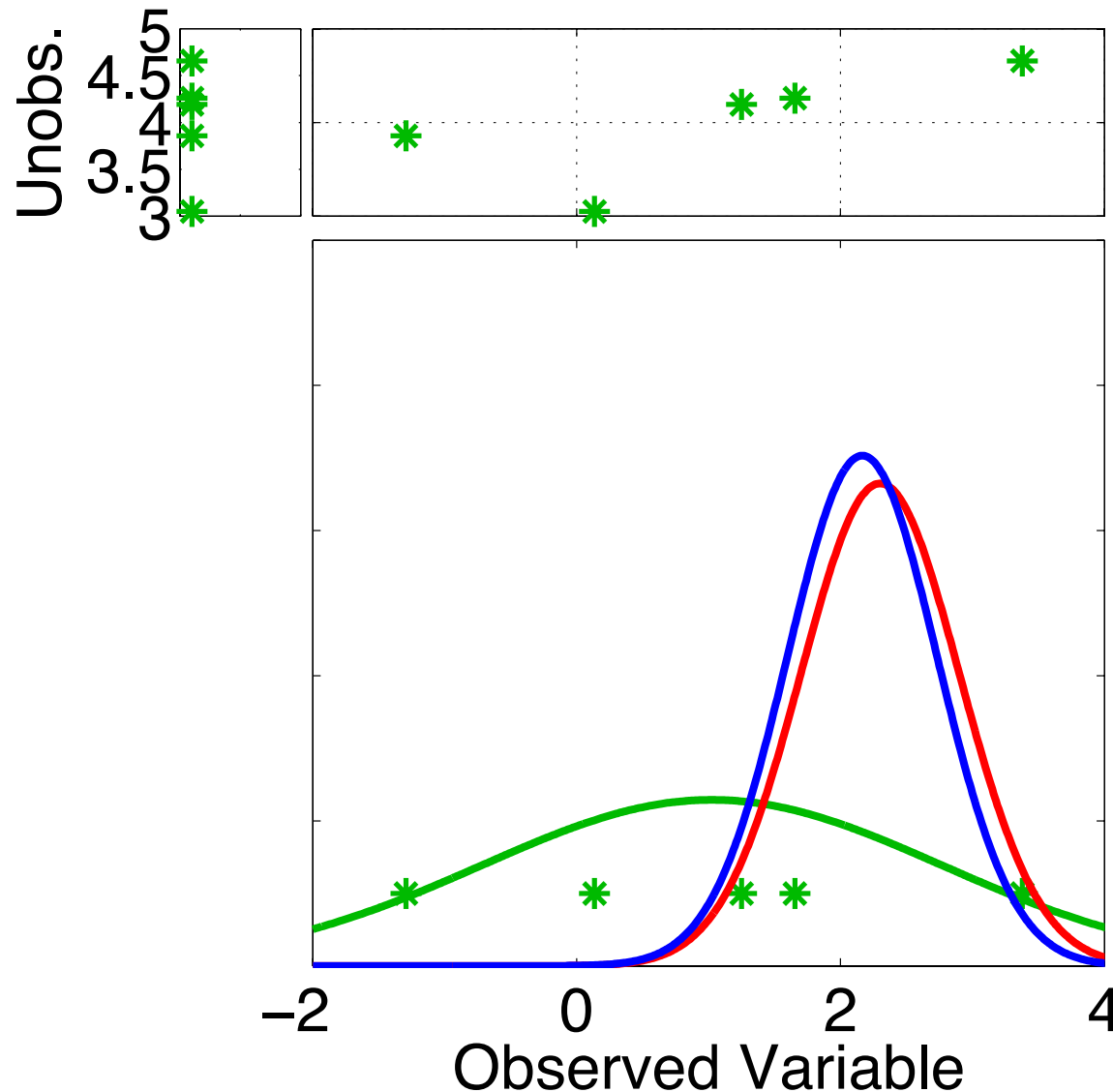


Assume that all we know is the prior joint distribution.

One variable is observed.

Update observed variable with ensemble Kalman filter.

Ensemble filters: Updating additional prior state variables

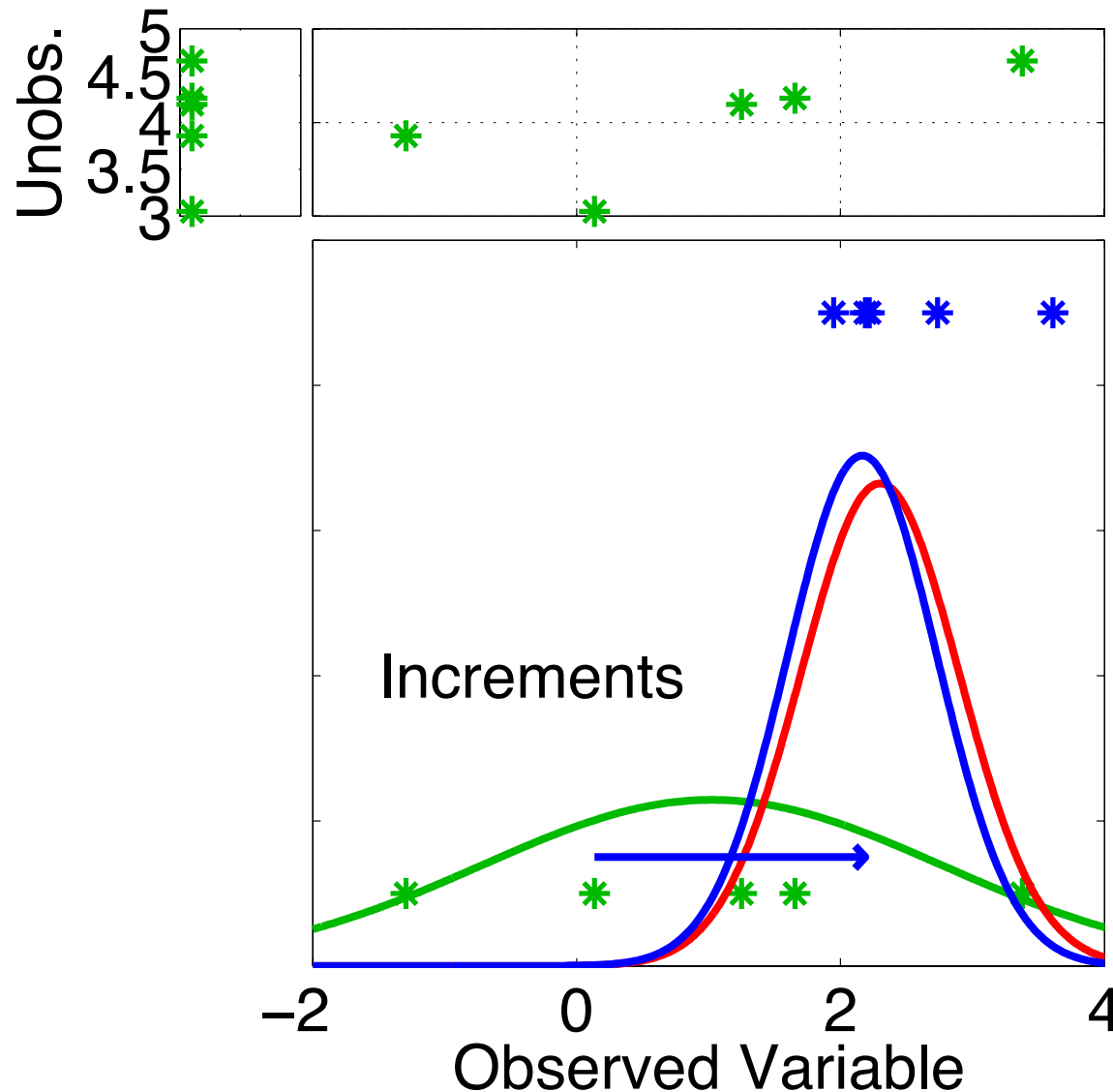


Assume that all we know is the prior joint distribution.

One variable is observed.

Update observed variable with ensemble Kalman filter.

Ensemble filters: Updating additional prior state variables

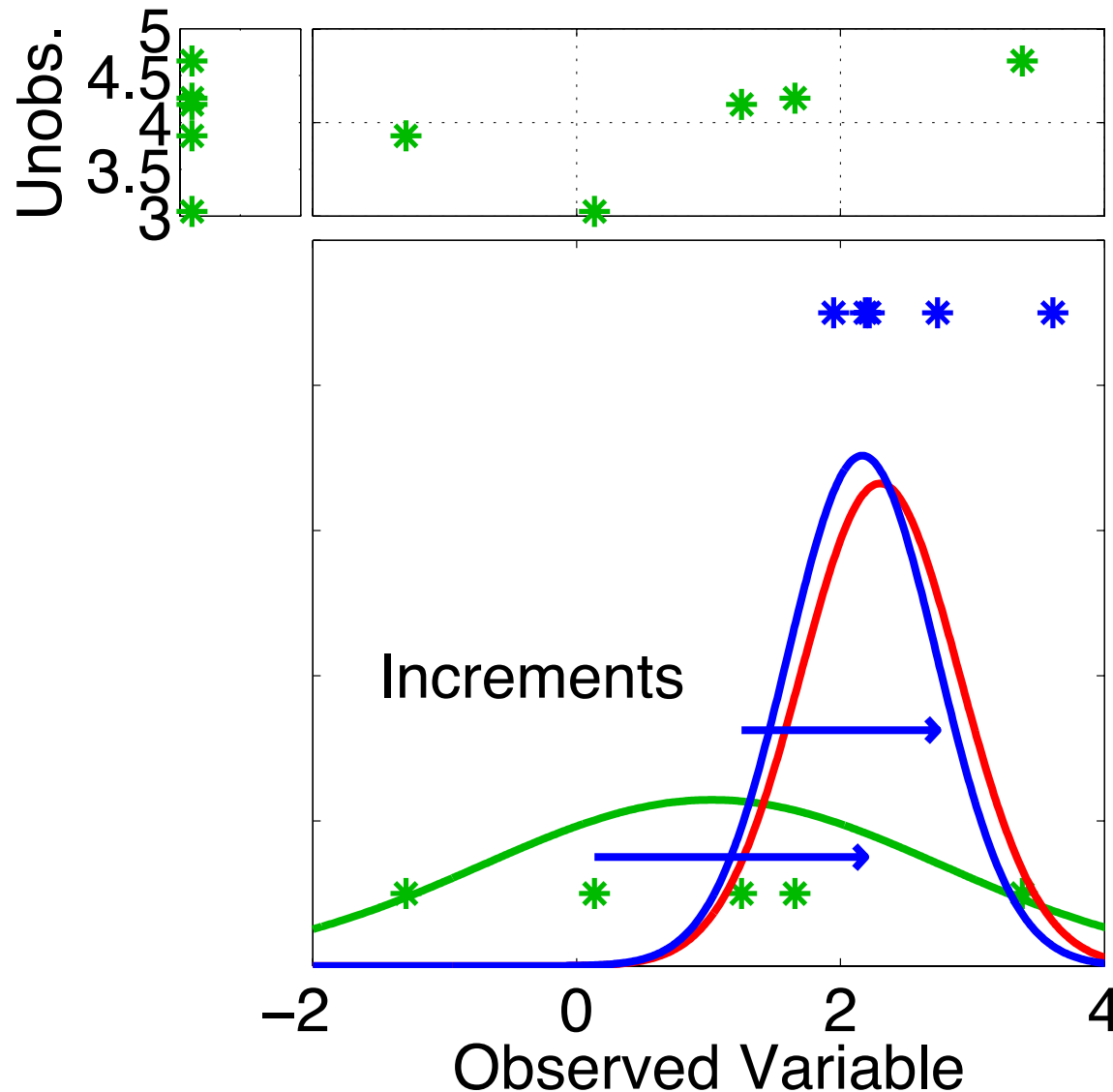


Assume that all we know is the prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.

Ensemble filters: Updating additional prior state variables

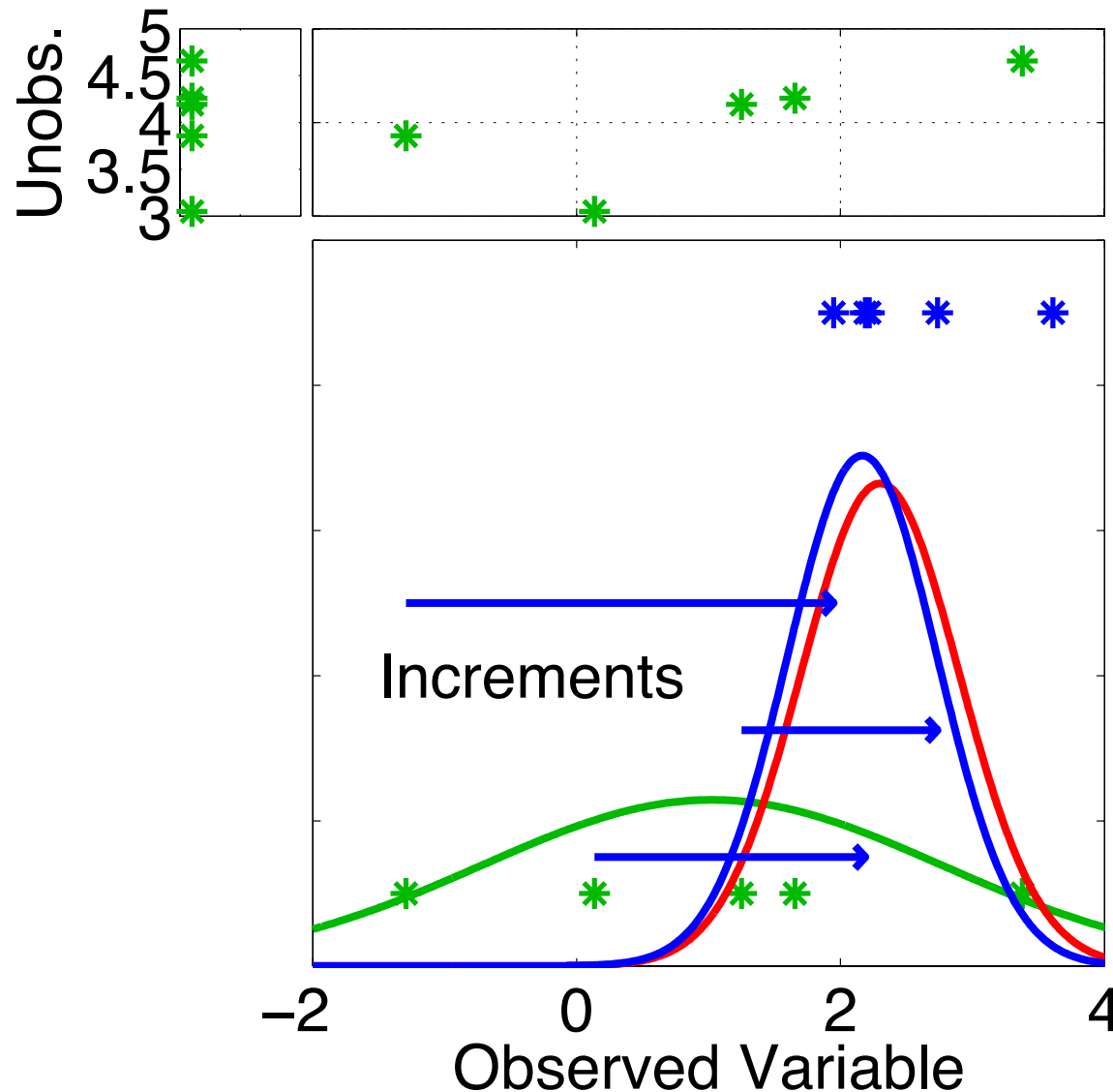


Assume that all we know is the prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.

Ensemble filters: Updating additional prior state variables

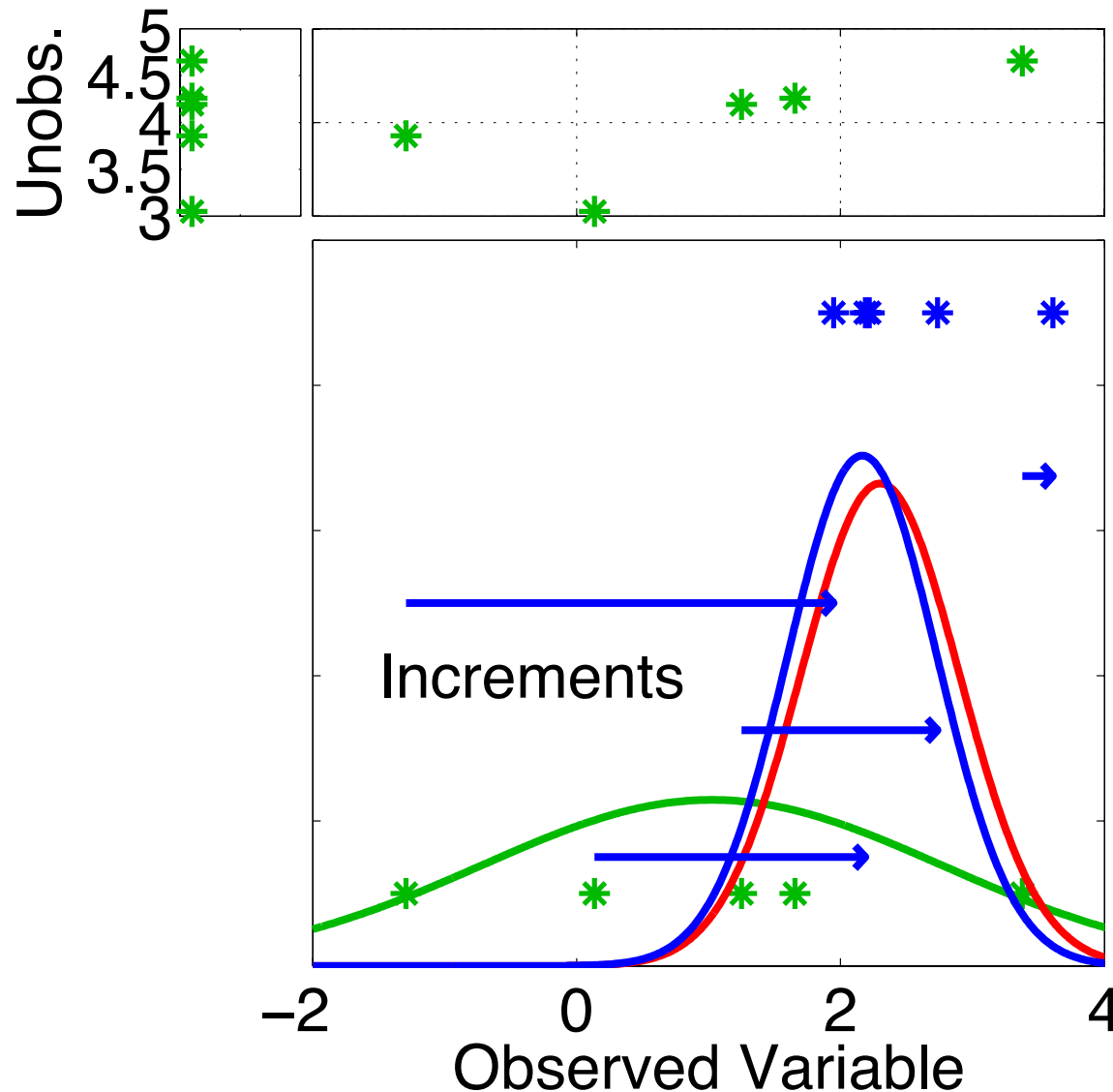


Assume that all we know is the prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.

Ensemble filters: Updating additional prior state variables

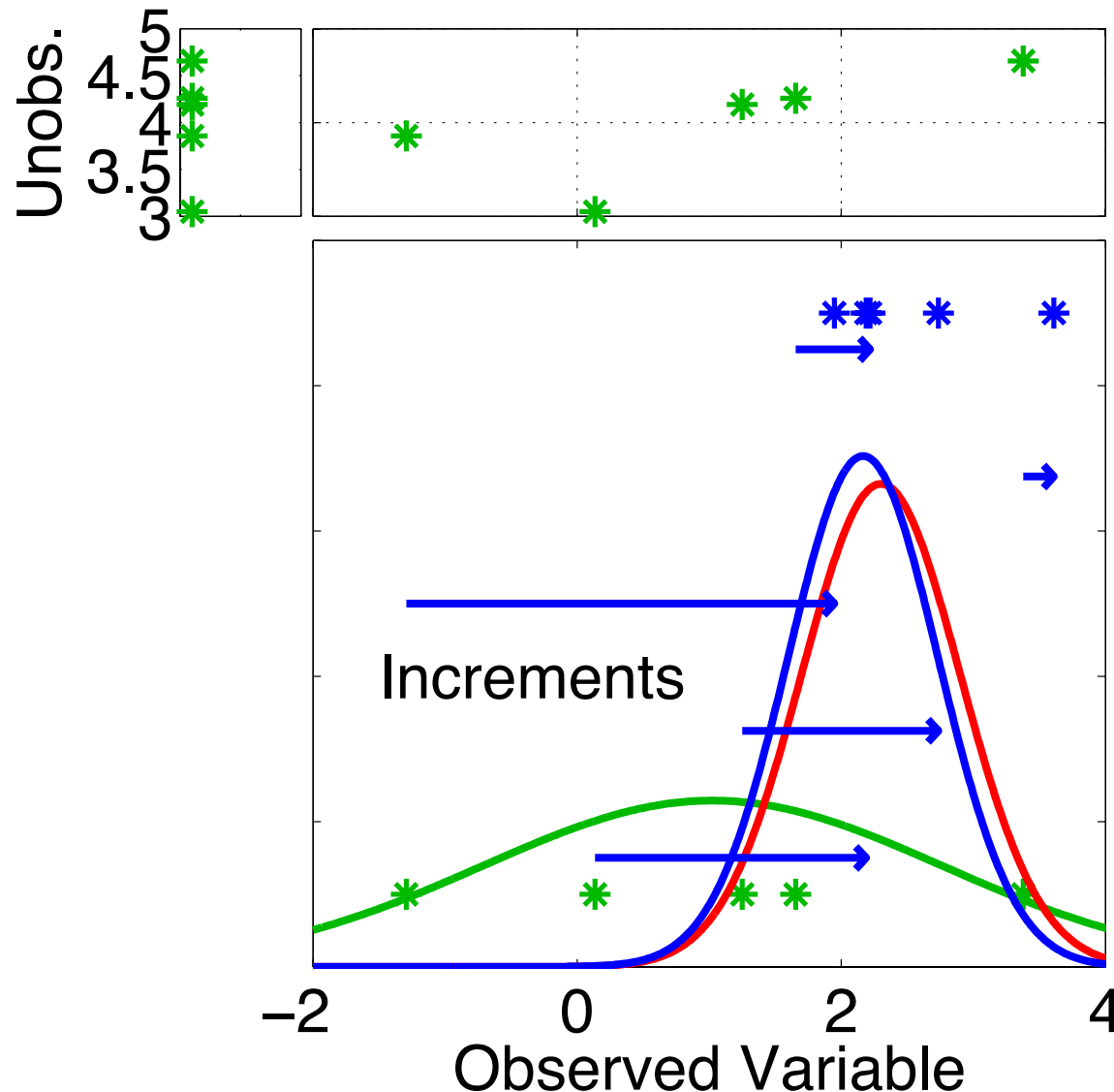


Assume that all we know is the prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.

Ensemble filters: Updating additional prior state variables

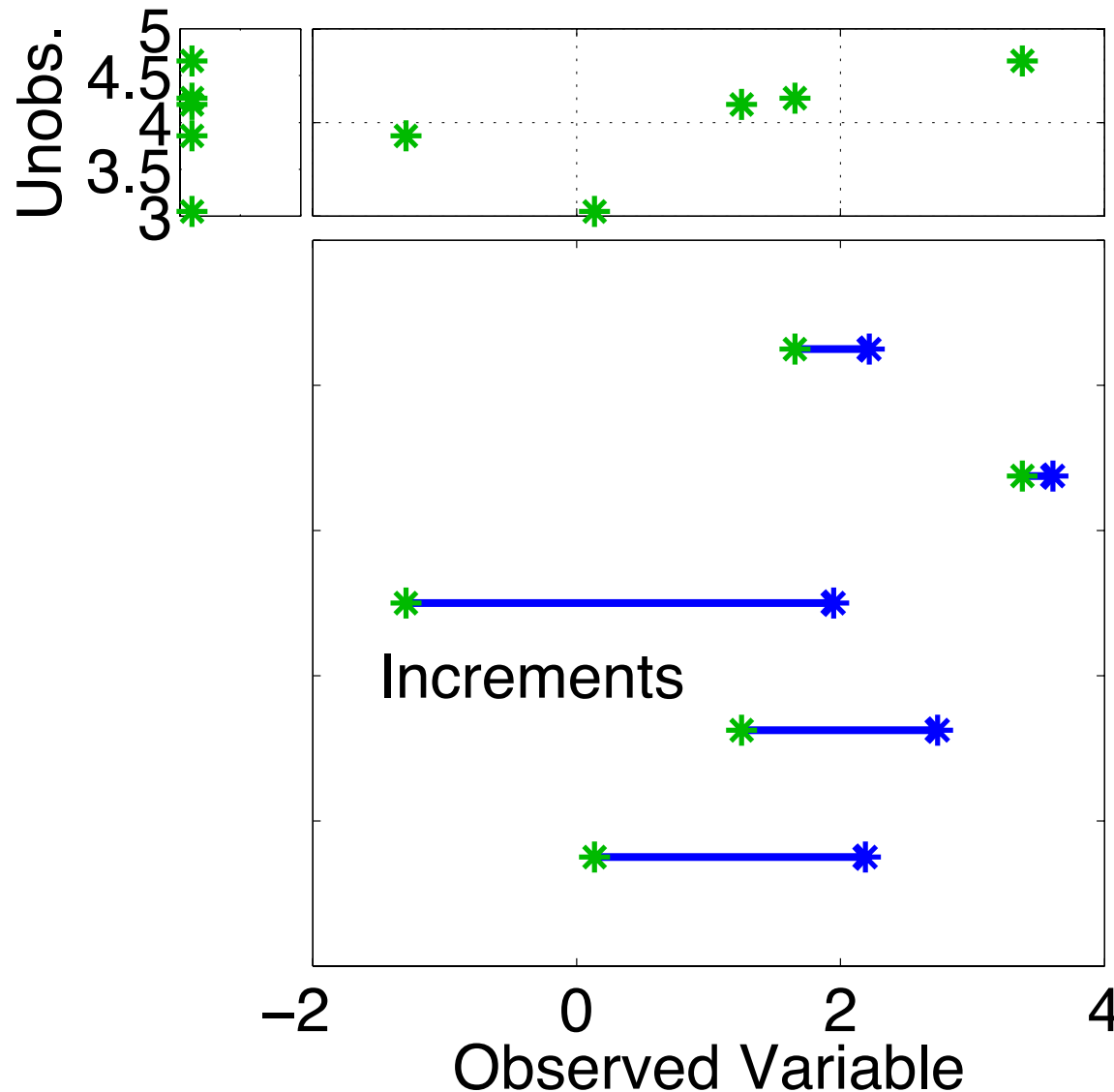


Assume that all we know is the prior joint distribution.

One variable is observed.

Compute increments for prior ensemble members of observed variable.

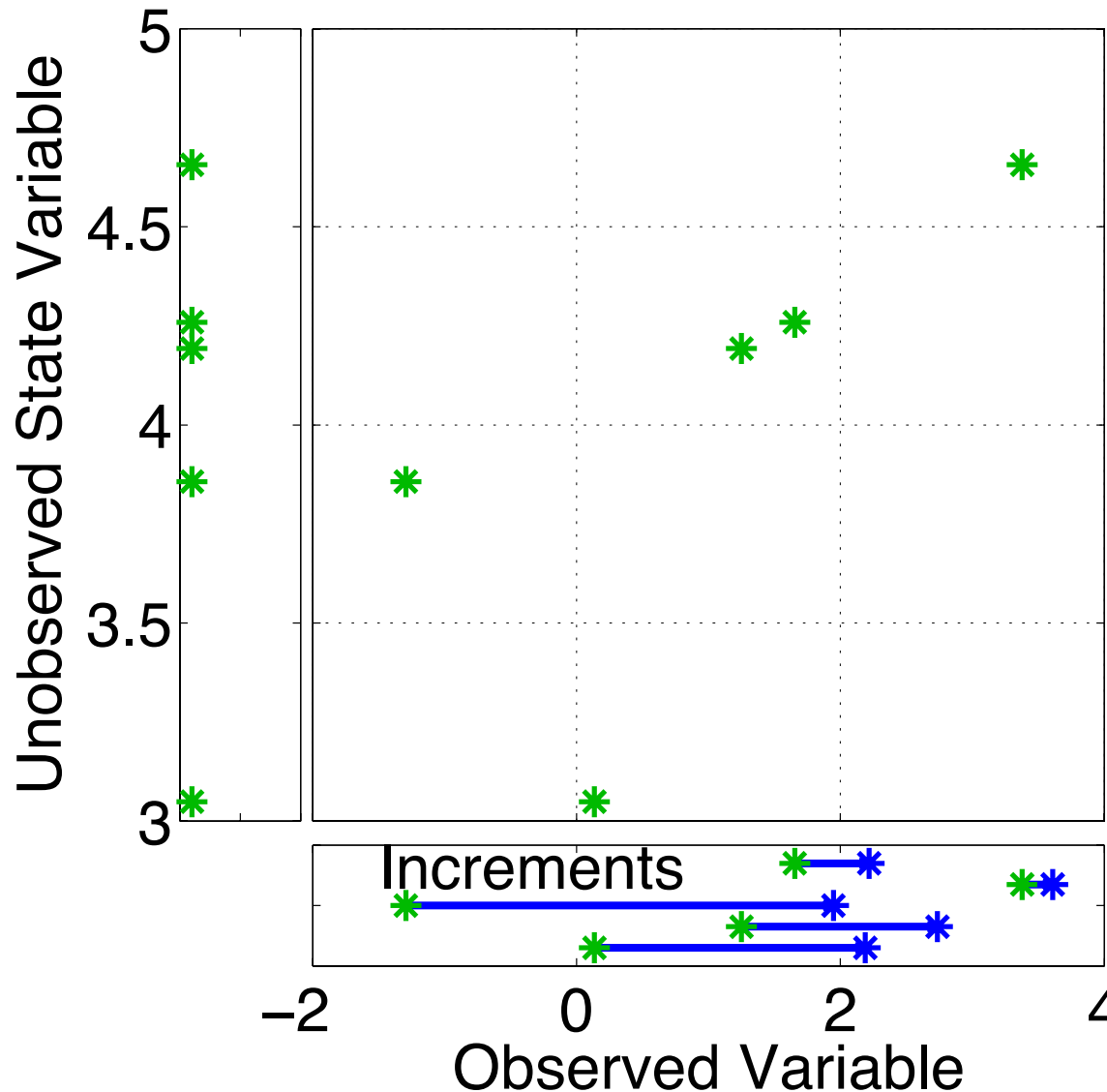
Ensemble filters: Updating additional prior state variables



Using only increments guarantees that if observation had no impact on observed variable, the unobserved variable is unchanged.

Highly desirable!

Ensemble filters: Updating additional prior state variables



Assume that all we know is the prior joint distribution.

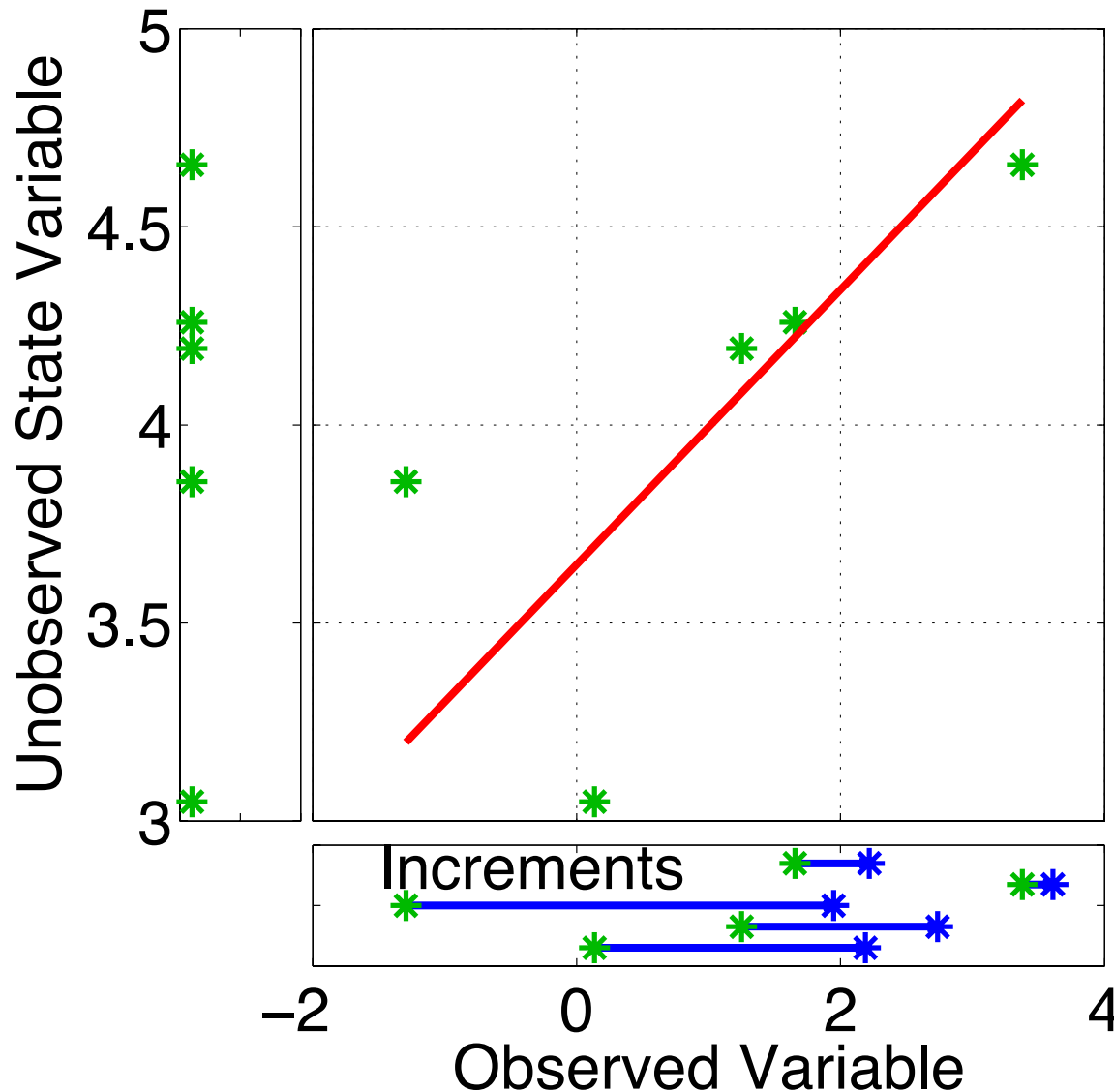
How should the unobserved variable be impacted?

1st choice: least squares

Equivalent to linear regression.

Same as assuming binormal prior.

Ensemble filters: Updating additional prior state variables



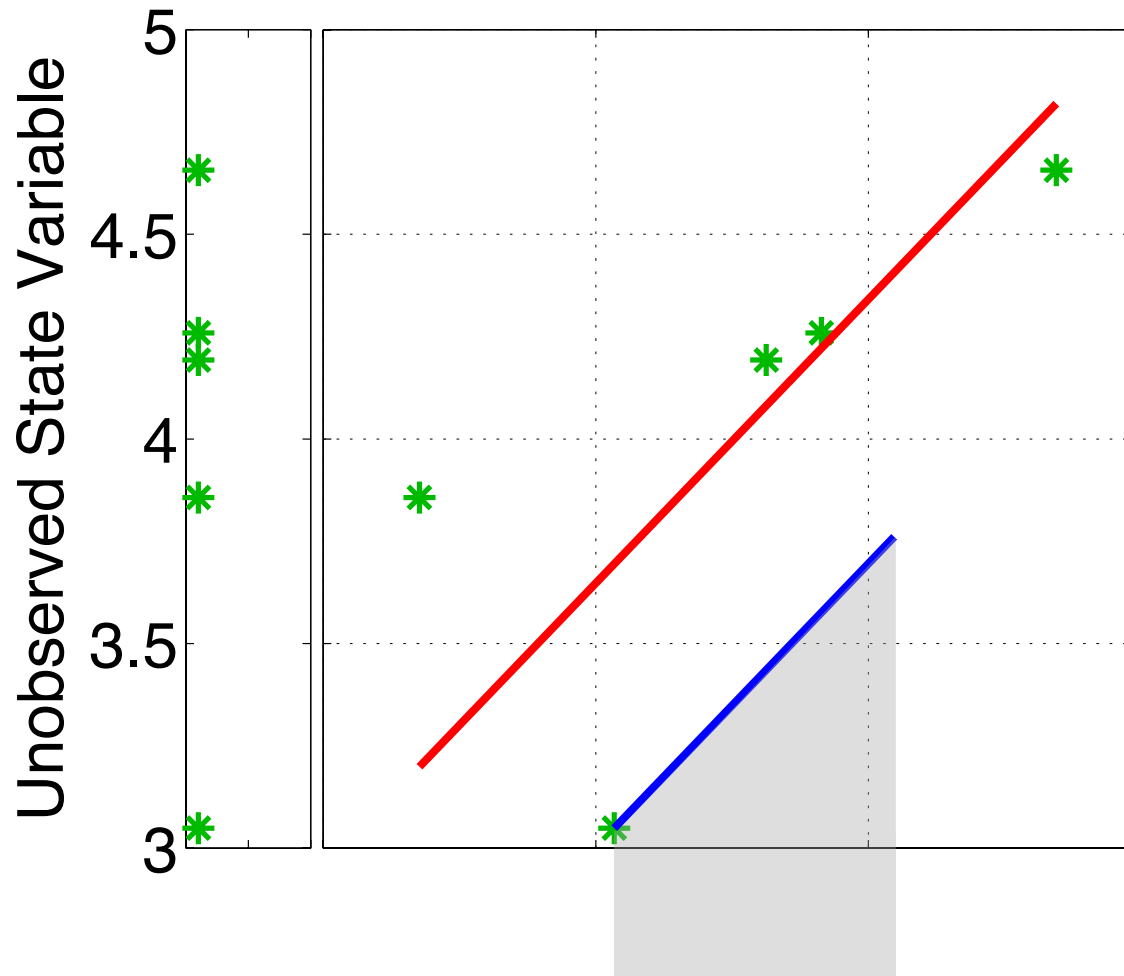
Have joint prior distribution of two variables.

How should the unobserved variable be impacted?

1st choice: least squares

Begin by finding **least squares fit**.

Ensemble filters: Updating additional prior state variables

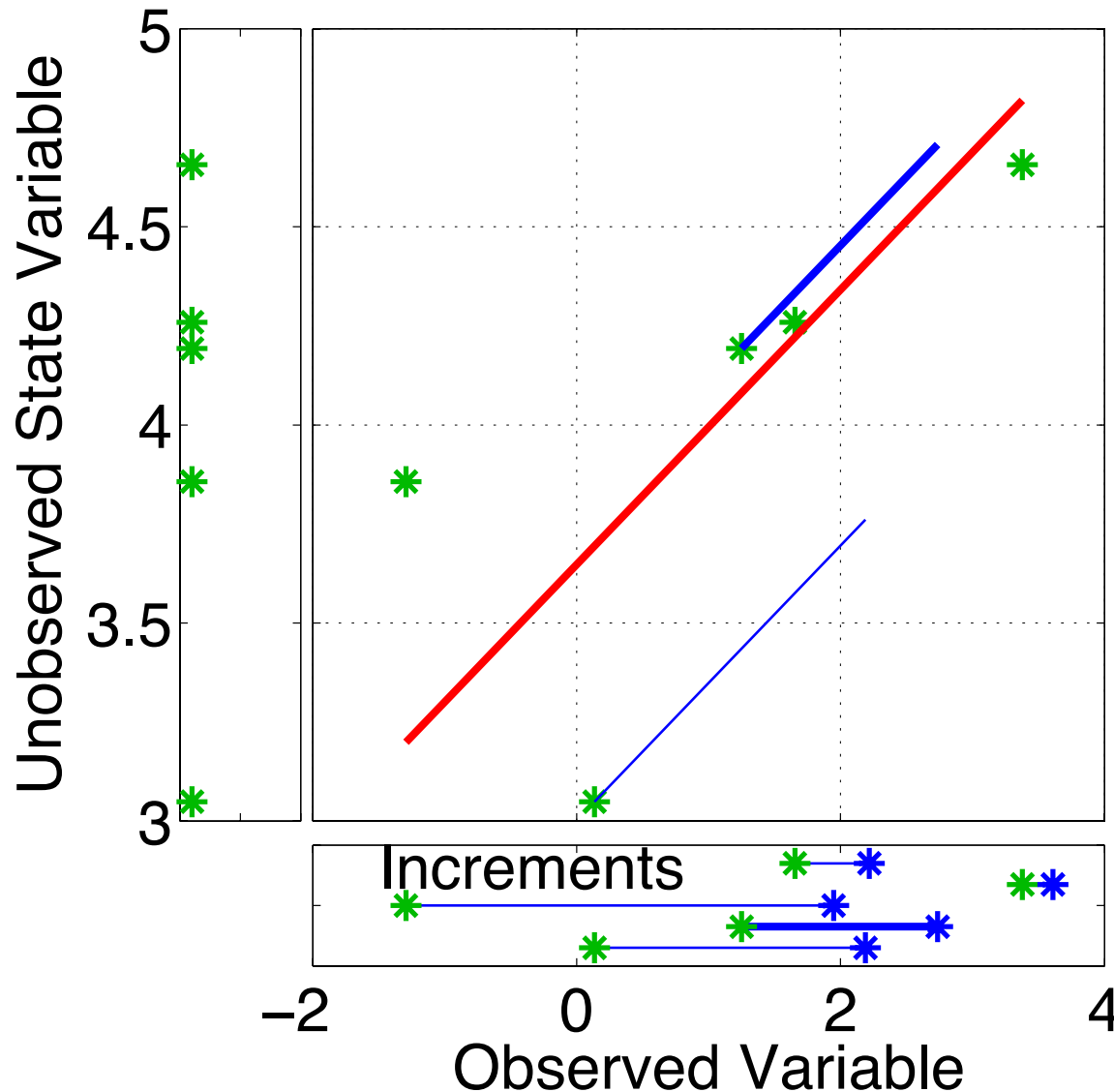


Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

Ensemble filters: Updating additional prior state variables

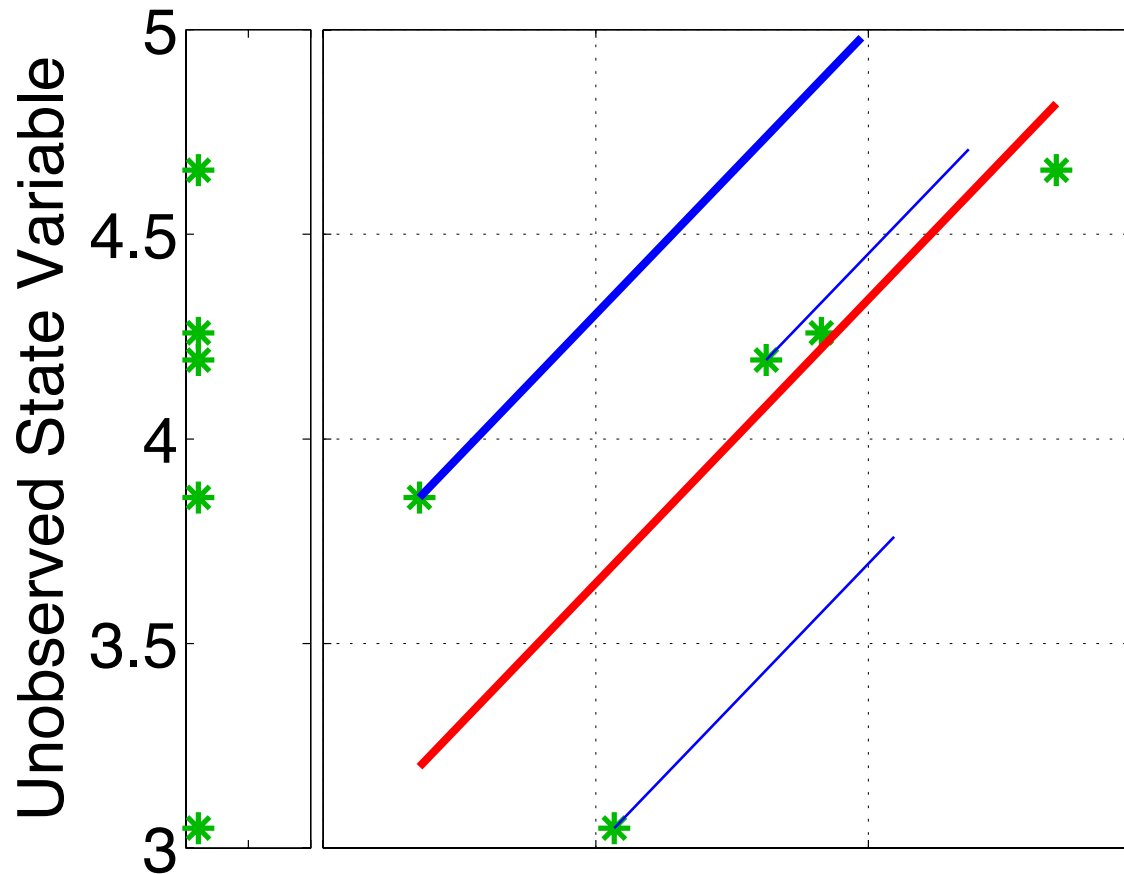


Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

Ensemble filters: Updating additional prior state variables

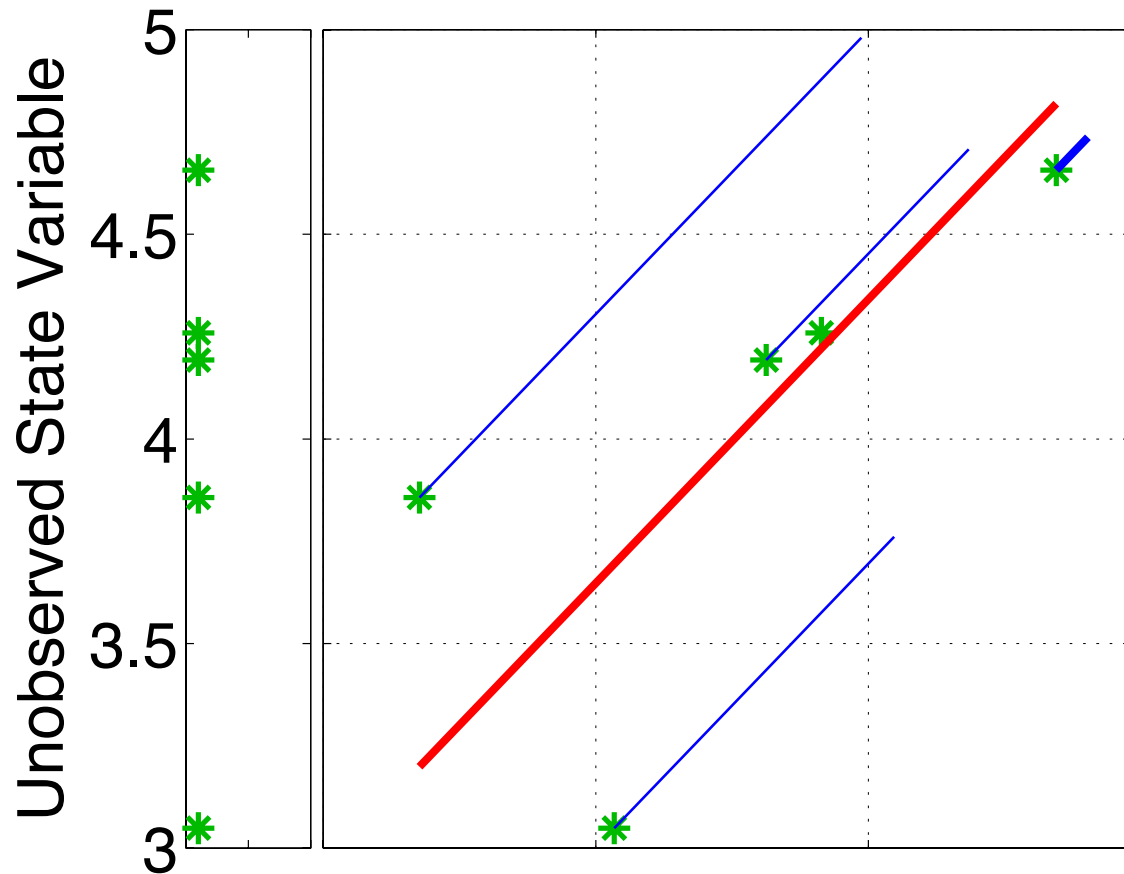


Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

Ensemble filters: Updating additional prior state variables

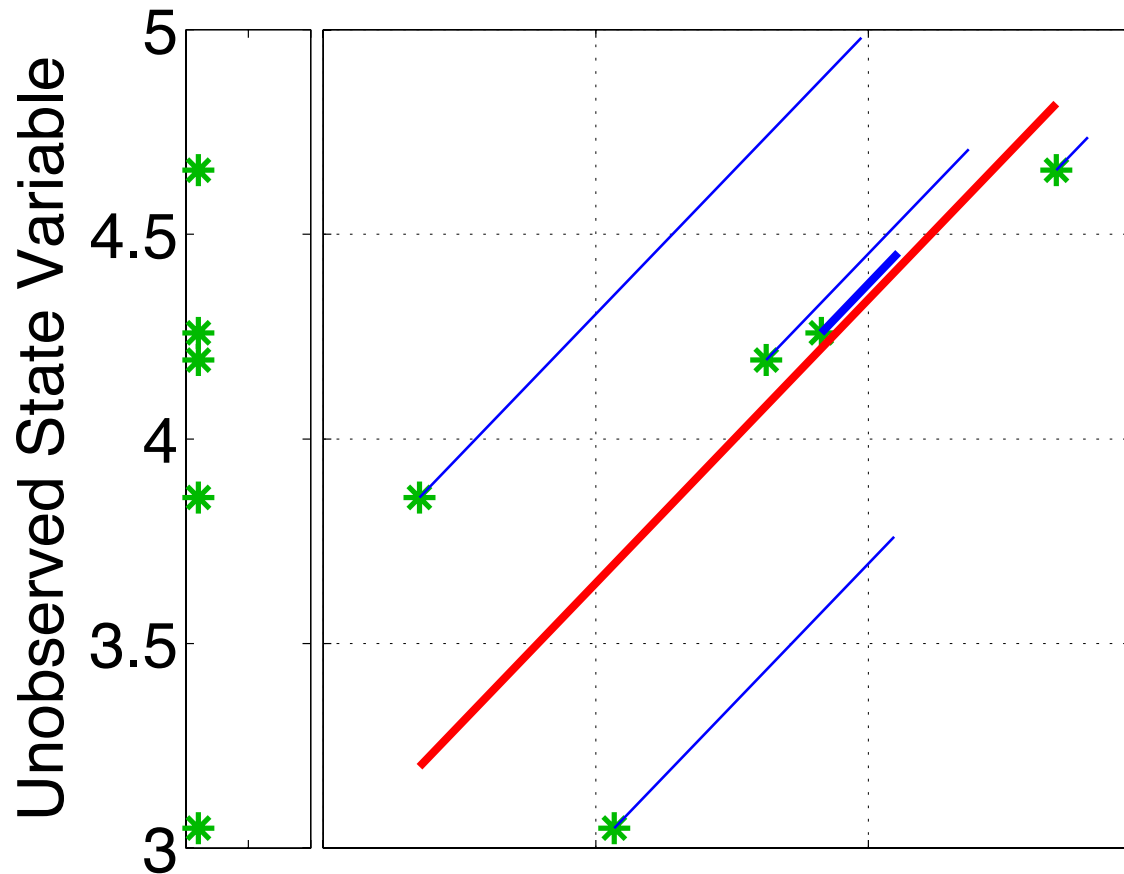


Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

Ensemble filters: Updating additional prior state variables

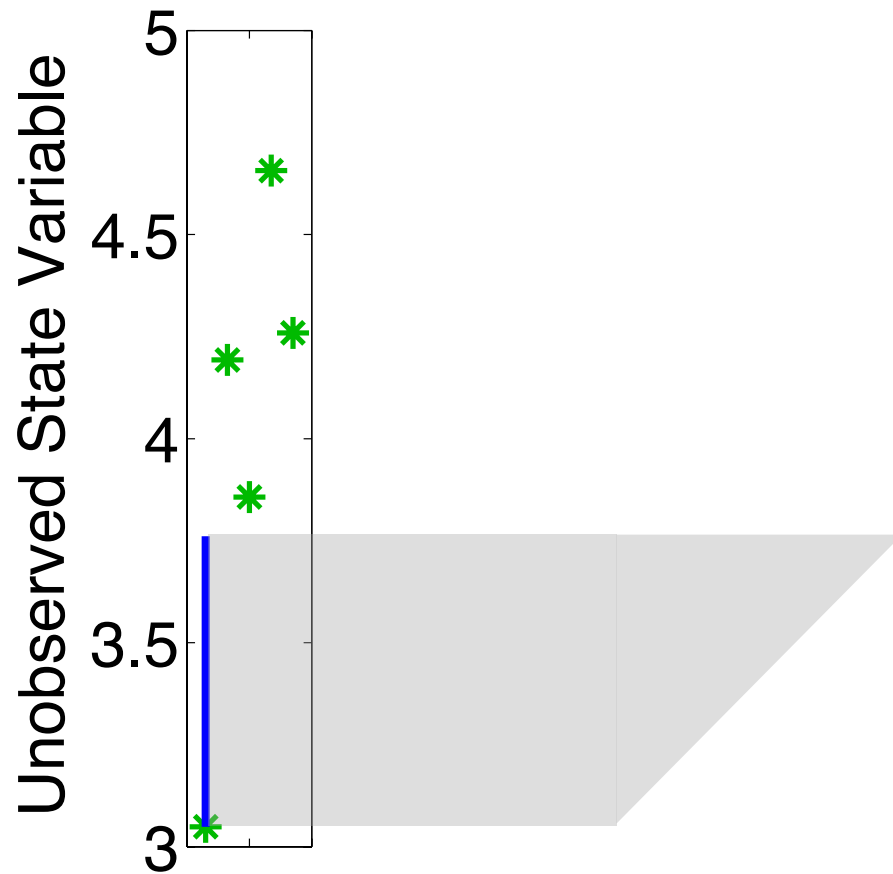


Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.

Ensemble filters: Updating additional prior state variables

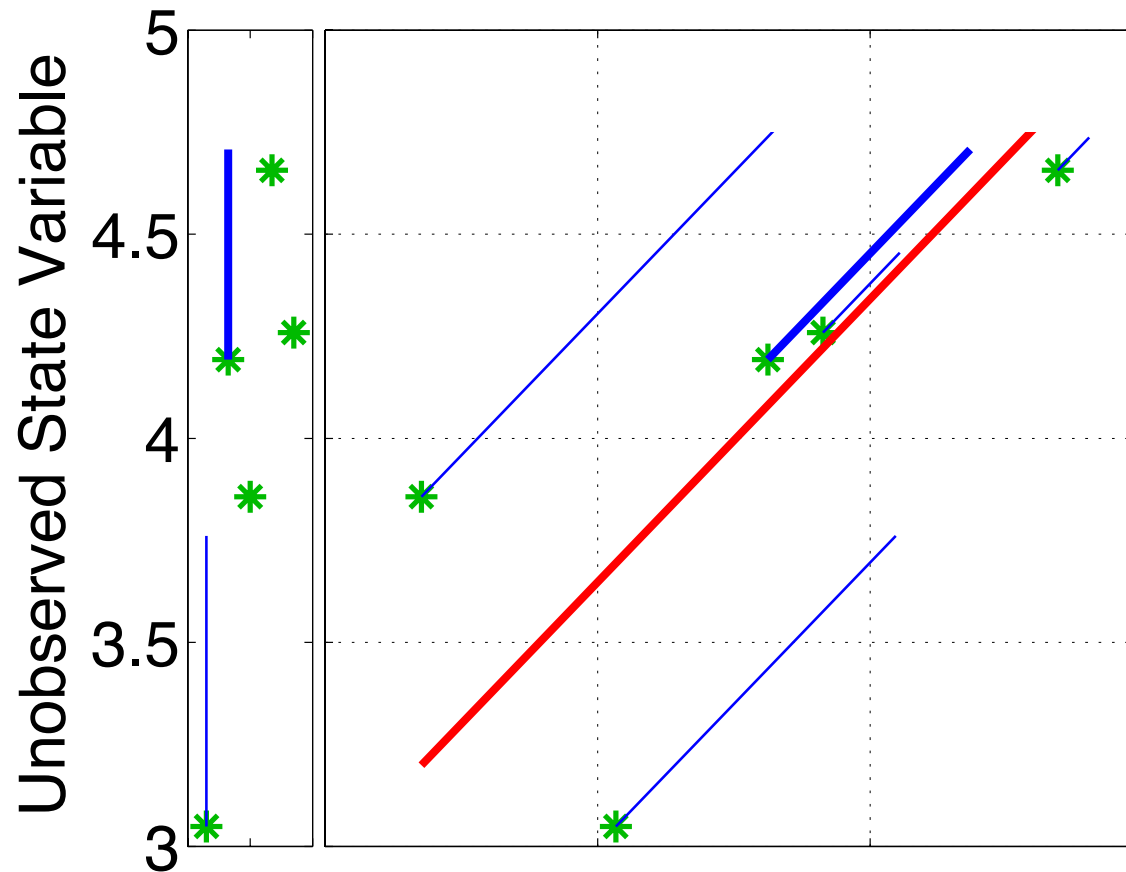


Have joint prior distribution of two variables.

Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.

Ensemble filters: Updating additional prior state variables

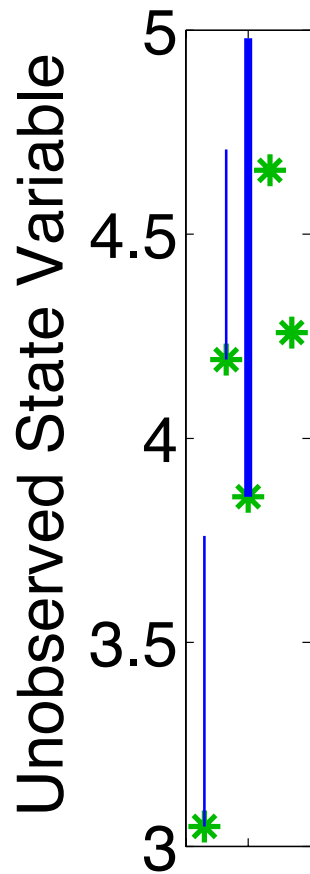


Have joint prior distribution of two variables.

Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.

Ensemble filters: Updating additional prior state variables

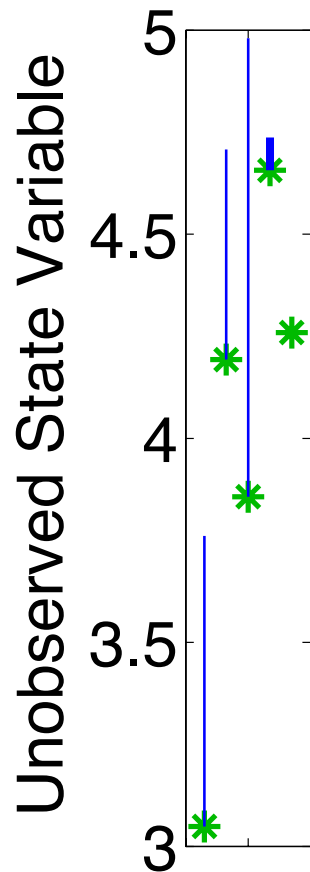


Have joint prior distribution of two variables.

Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.

Ensemble filters: Updating additional prior state variables

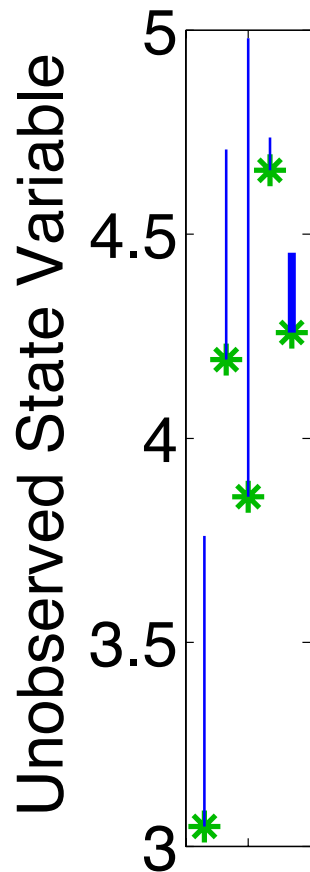


Have joint prior distribution of two variables.

Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.

Ensemble filters: Updating additional prior state variables

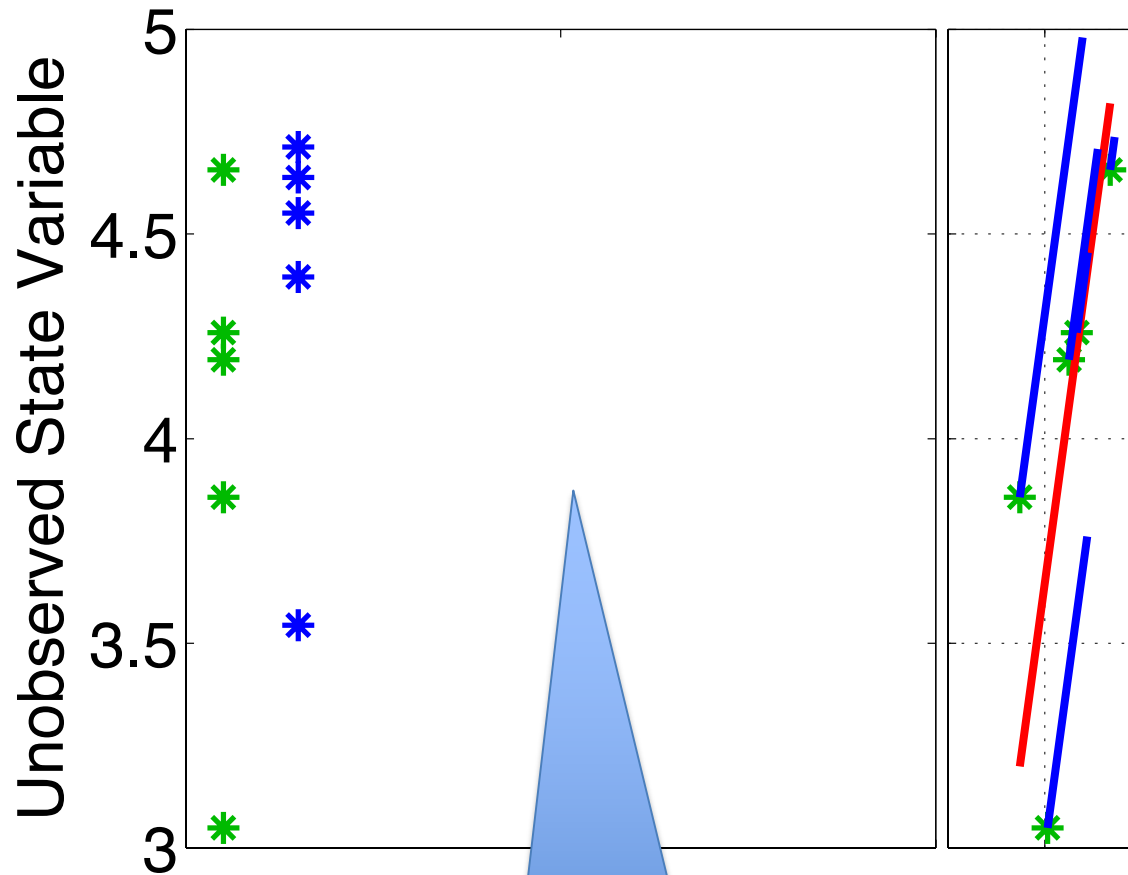


Have joint prior distribution of two variables.

Regression: Equivalent to first finding image of increment in joint space.

Then projecting from joint space onto unobserved priors.

Ensemble filters: Updating additional prior state variables

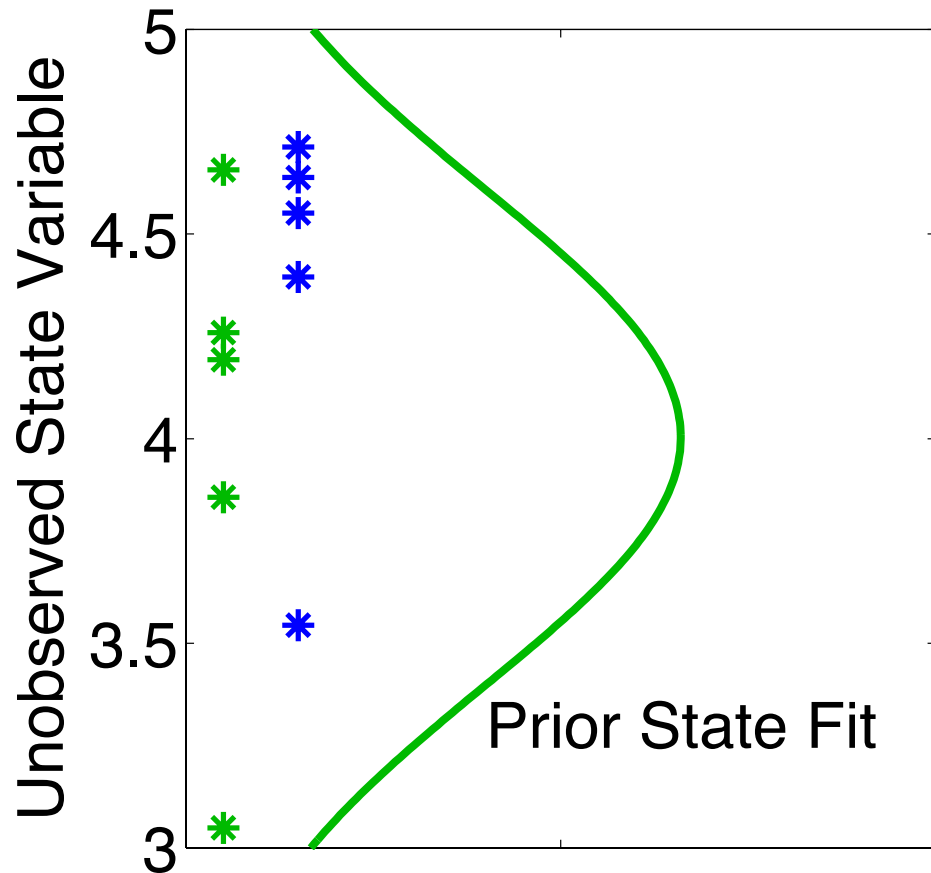


Now have an updated (posterior) ensemble for the unobserved variable.

We've expanded this plot. Same information as previous slides.

Compressed these two.

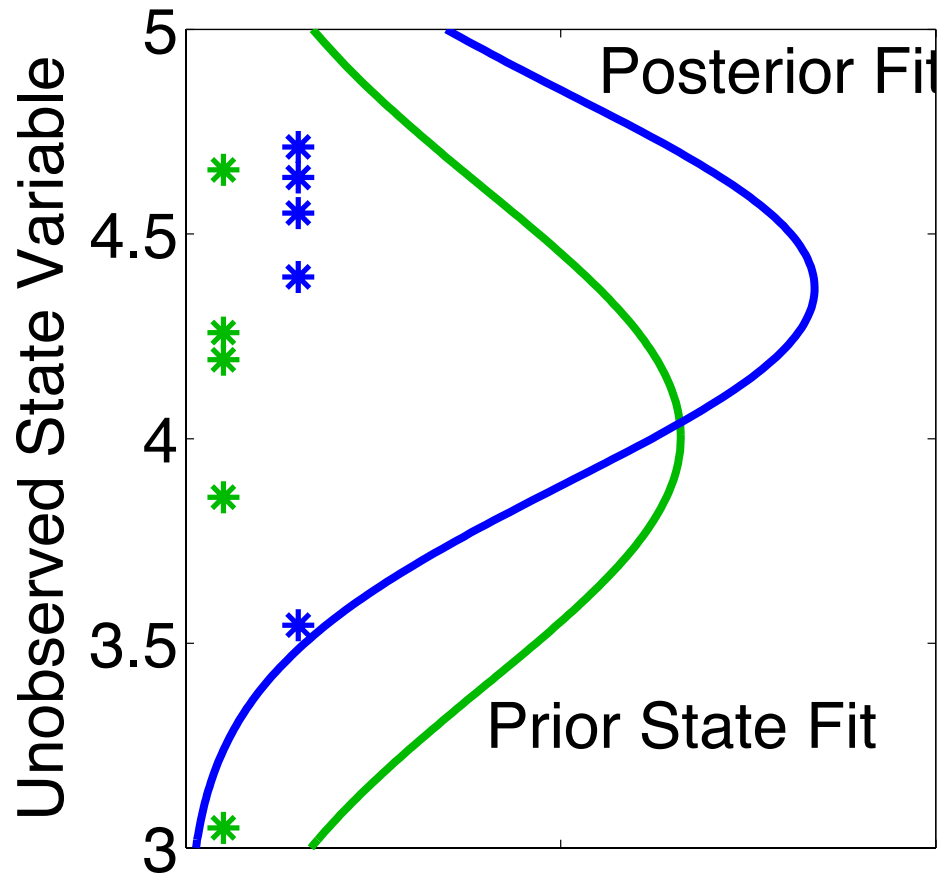
Ensemble filters: Updating additional prior state variables



Now have an updated (posterior) ensemble for the unobserved variable.

Fitting Gaussians shows that mean and variance have changed.

Ensemble filters: Updating additional prior state variables



Now have an updated (posterior) ensemble for the unobserved variable.

Fitting Gaussians shows that mean and variance have changed.

Other features of the prior distribution may also have changed.

Properties of Ensemble Kalman Filter

For linear, gaussian problem:

If, ensemble size $N > N_{\text{crit}}$

Mean and covariance are identical to Kalman Filter,

Else

Diverges.

N_{crit} : Number of positive singular values in SVD of covariance matrix.

How an Ensemble Filter Works for Geophysical Data Assimilation

1. Use model to advance **ensemble** (3 members here) to time at which next observation becomes available.

Ensemble state
estimate after using
previous observation
(analysis)

t_k



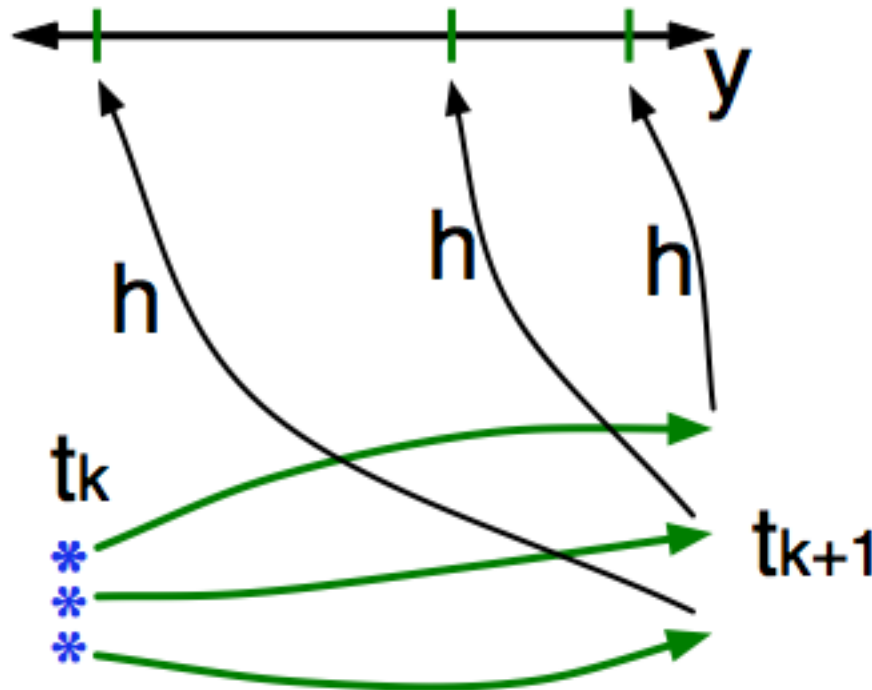
Ensemble state
at time of next
observation
(prior)

t_{k+1}



How an Ensemble Filter Works for Geophysical Data Assimilation

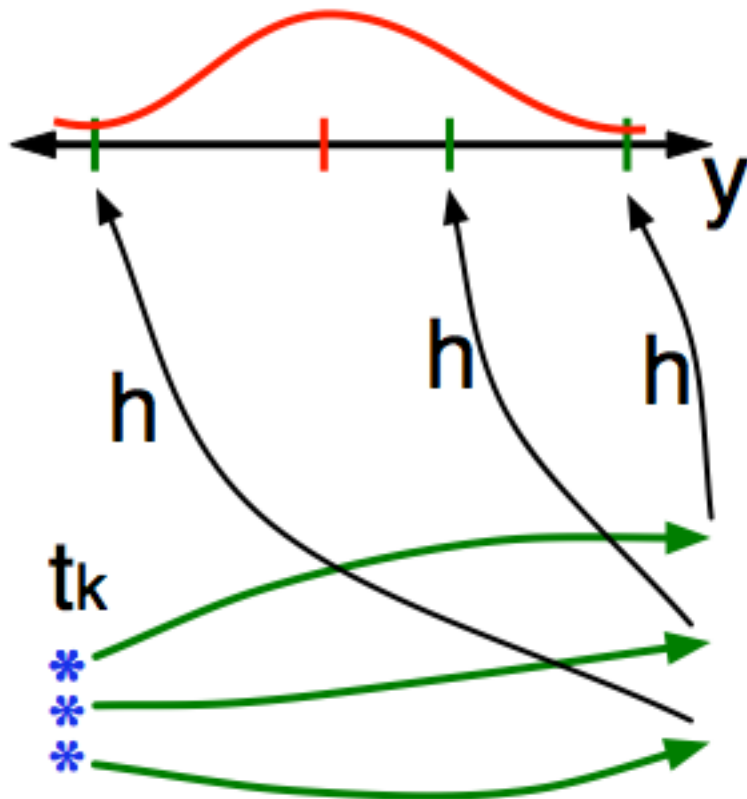
2. Get prior ensemble sample of observation, $y = h(x)$, by applying forward operator h to each ensemble member.



Theory: observations from instruments with uncorrelated errors can be done sequentially.

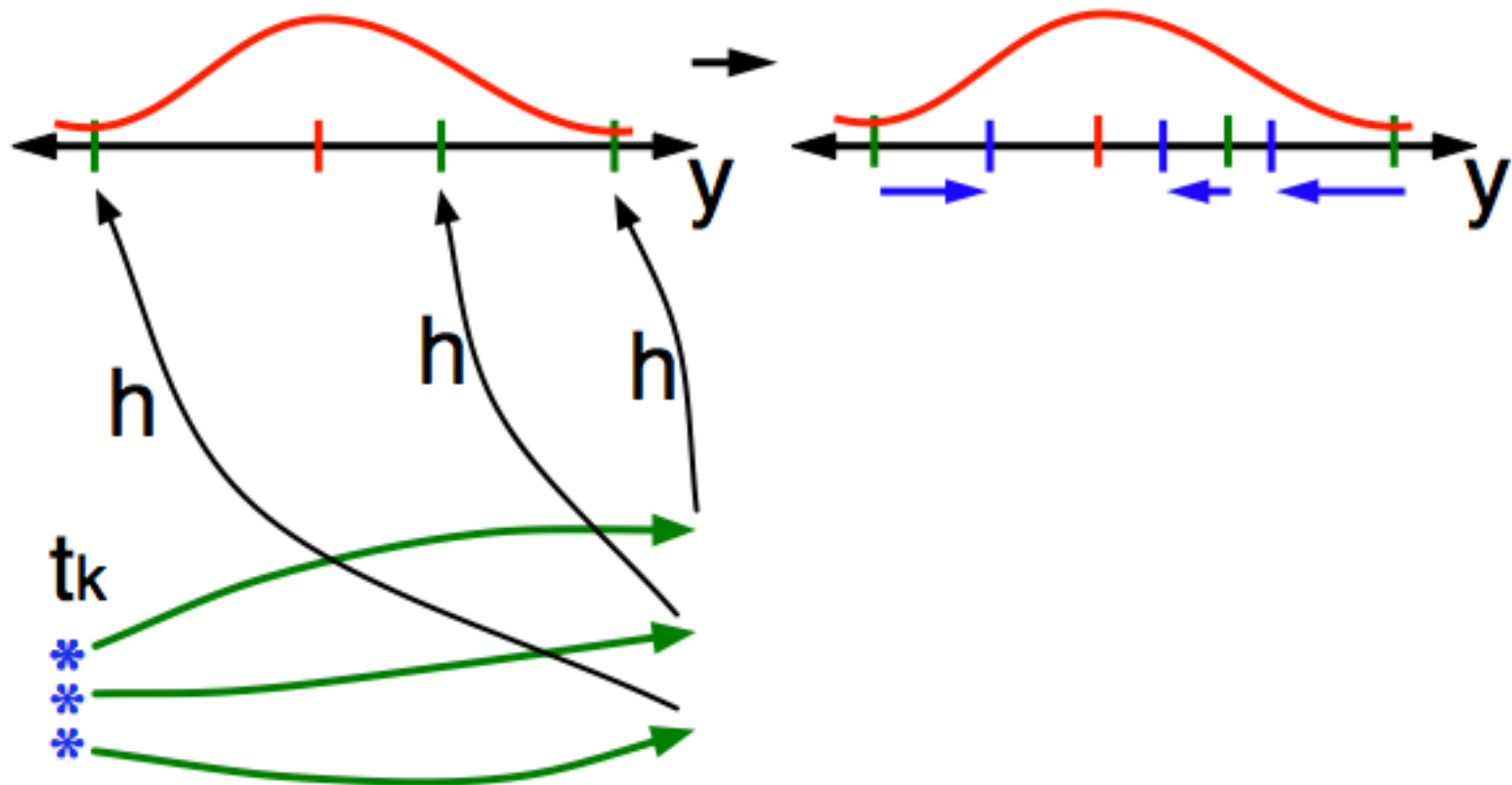
How an Ensemble Filter Works for Geophysical Data Assimilation

3. Get **observed value** and **observational error distribution** from observing system.



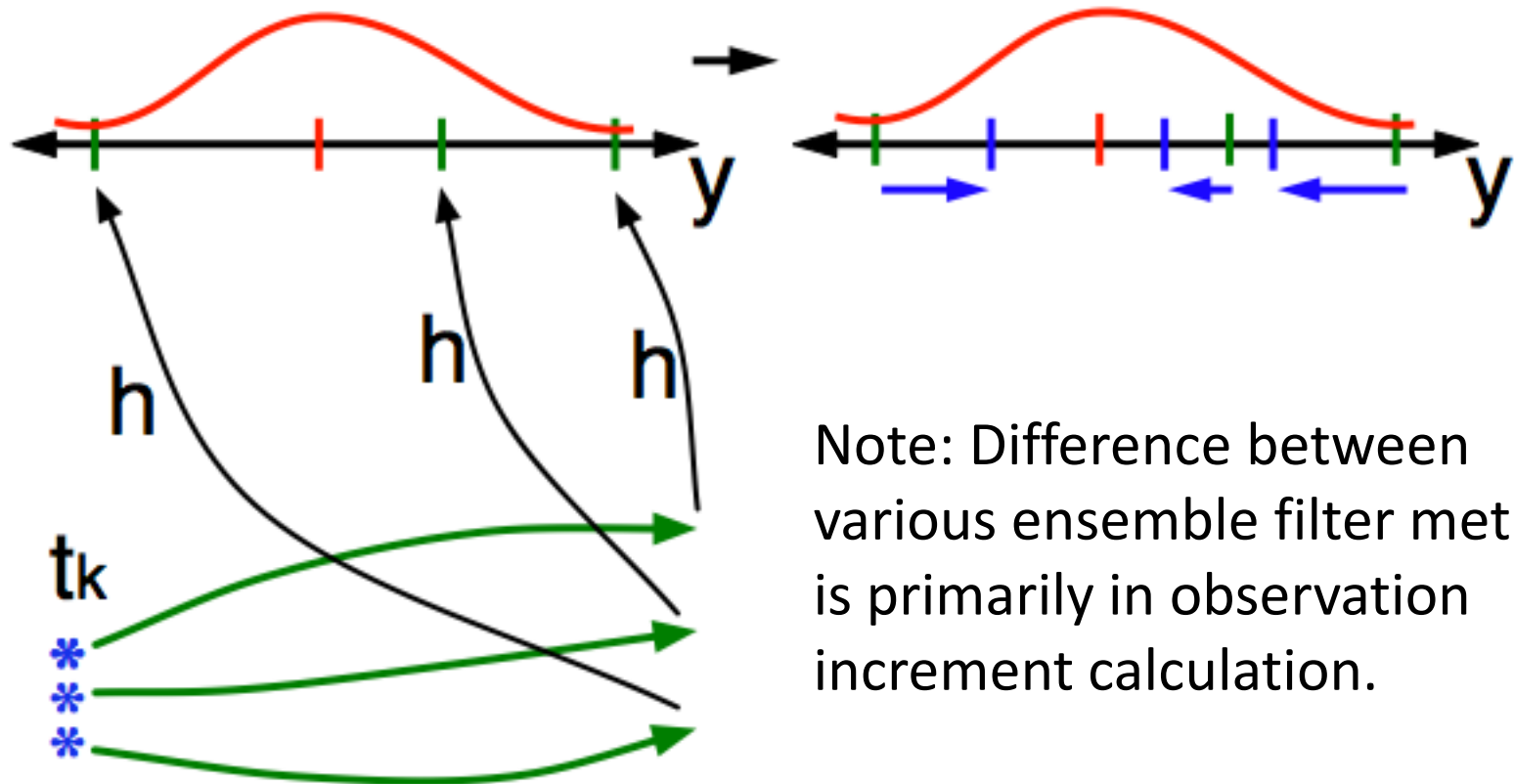
How an Ensemble Filter Works for Geophysical Data Assimilation

- Find the **increments** for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



How an Ensemble Filter Works for Geophysical Data Assimilation

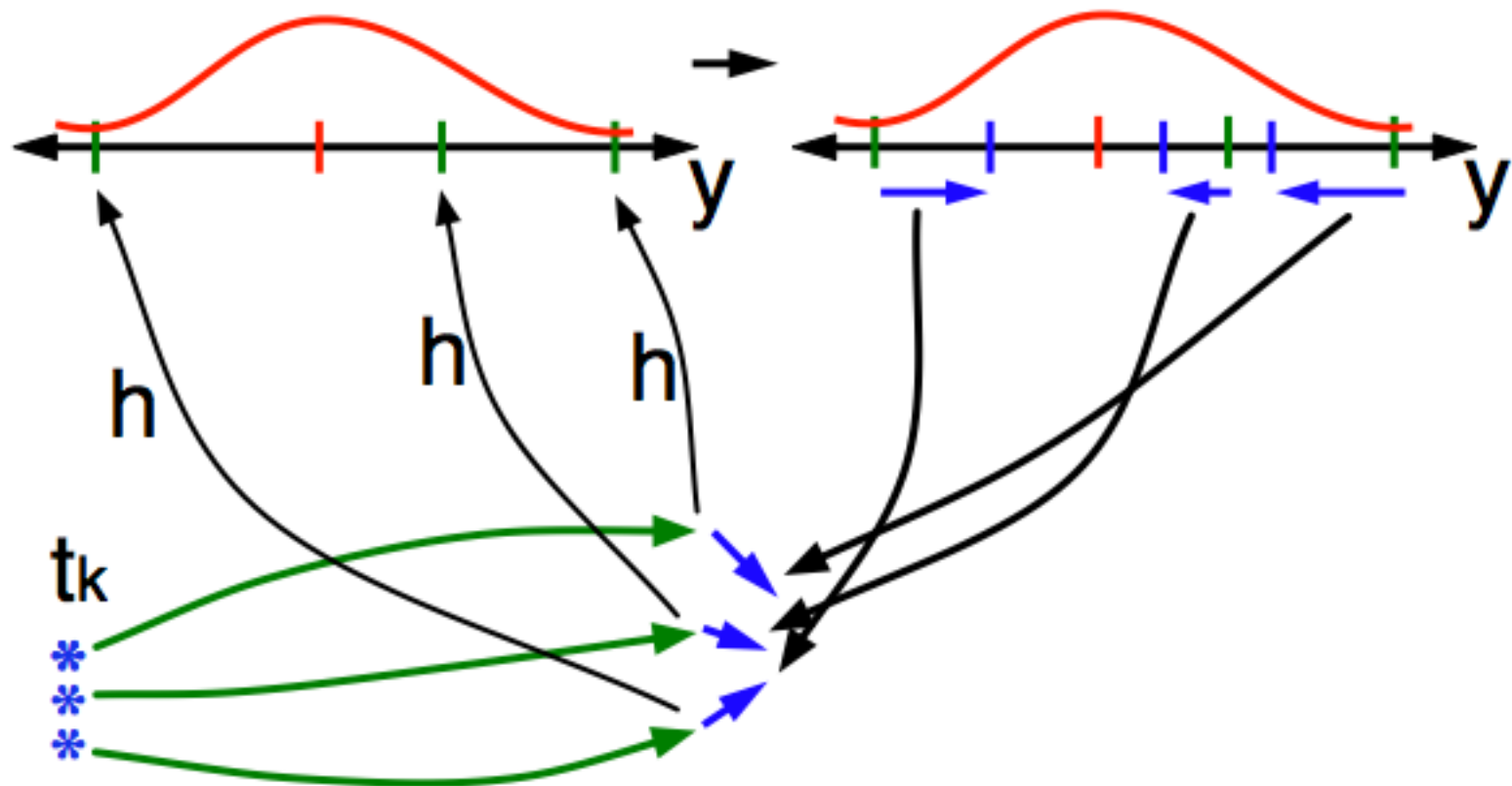
- Find the **increments** for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



Note: Difference between various ensemble filter methods is primarily in observation increment calculation.

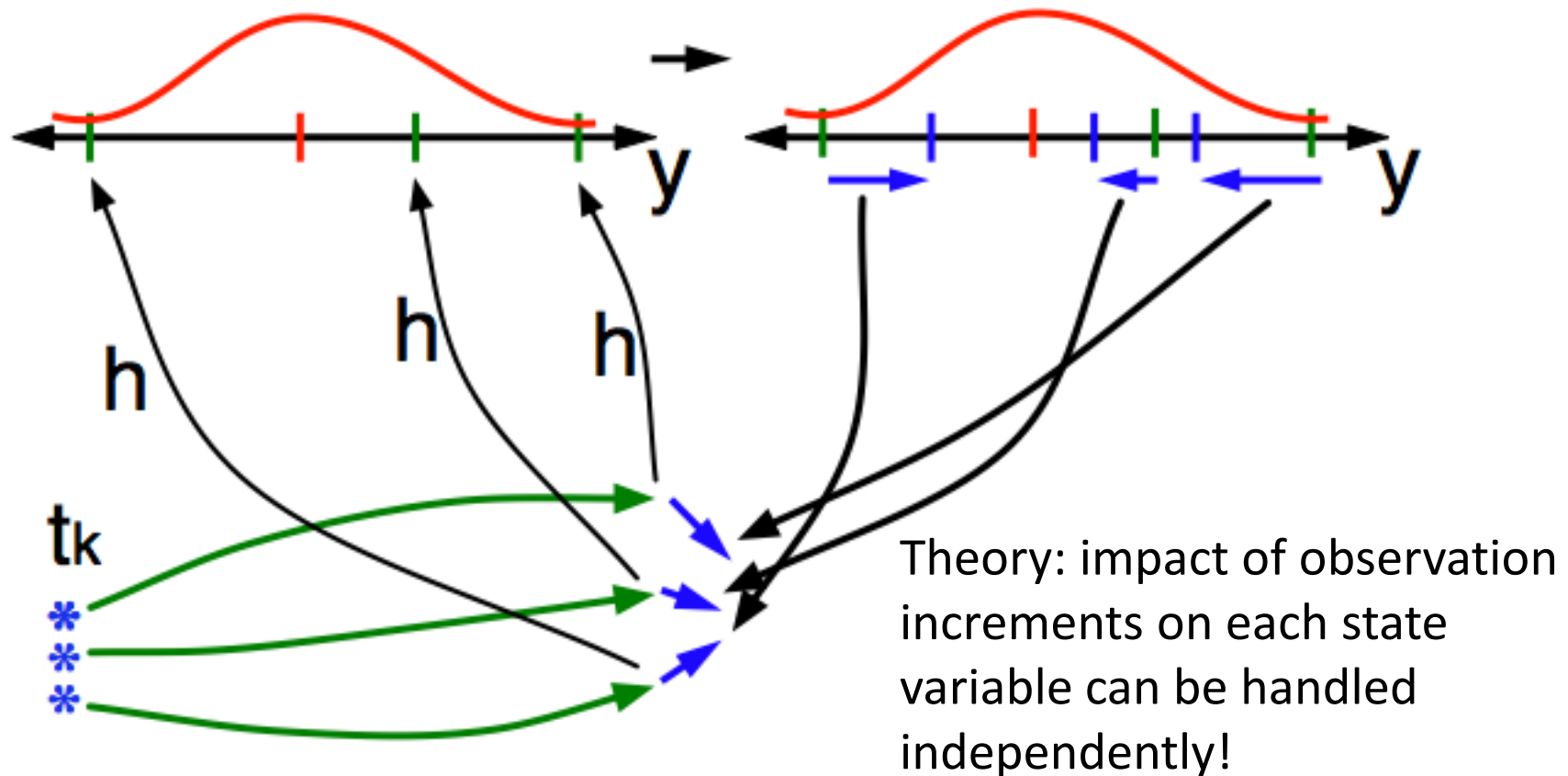
How an Ensemble Filter Works for Geophysical Data Assimilation

5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



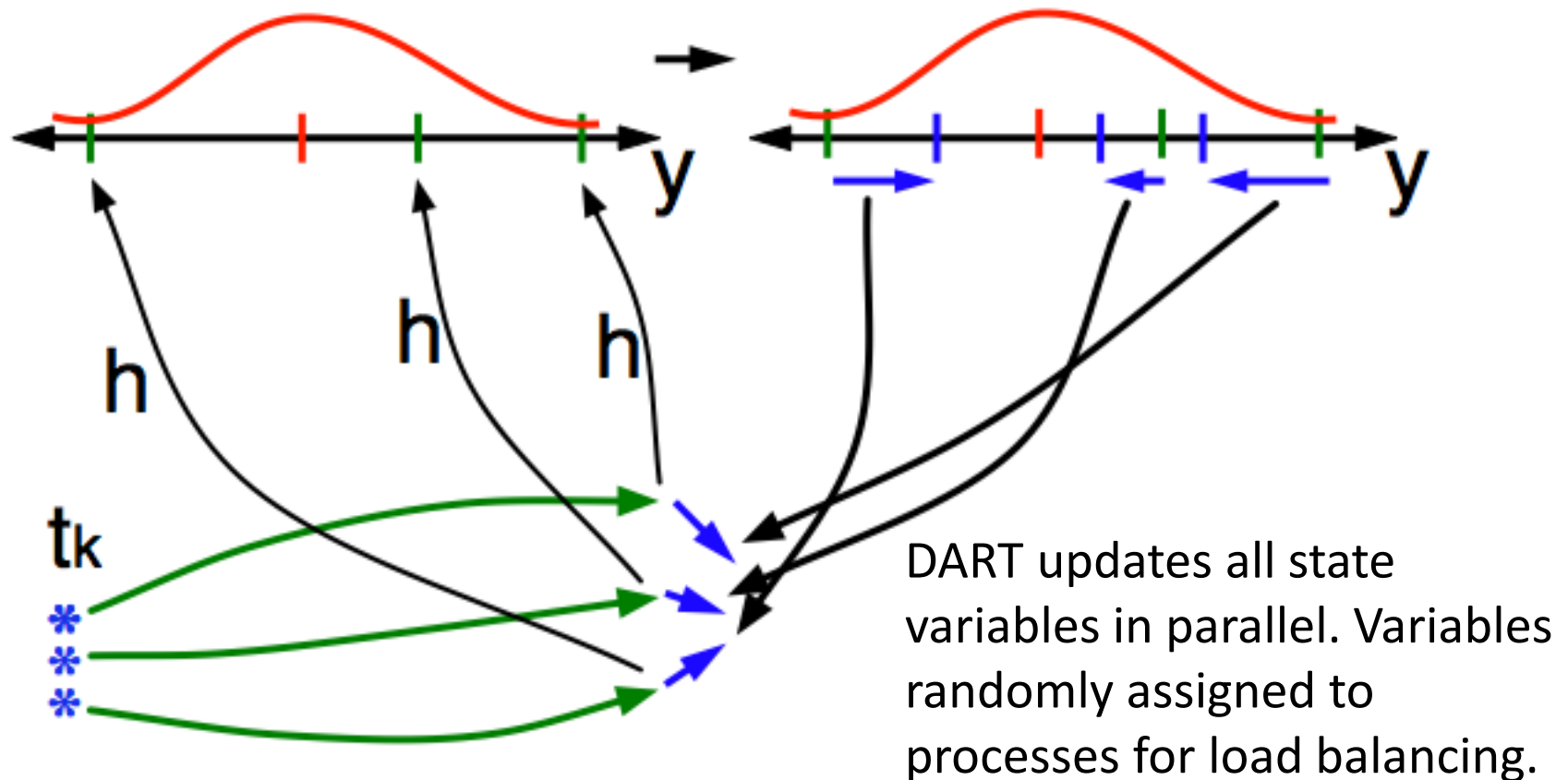
How an Ensemble Filter Works for Geophysical Data Assimilation

5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



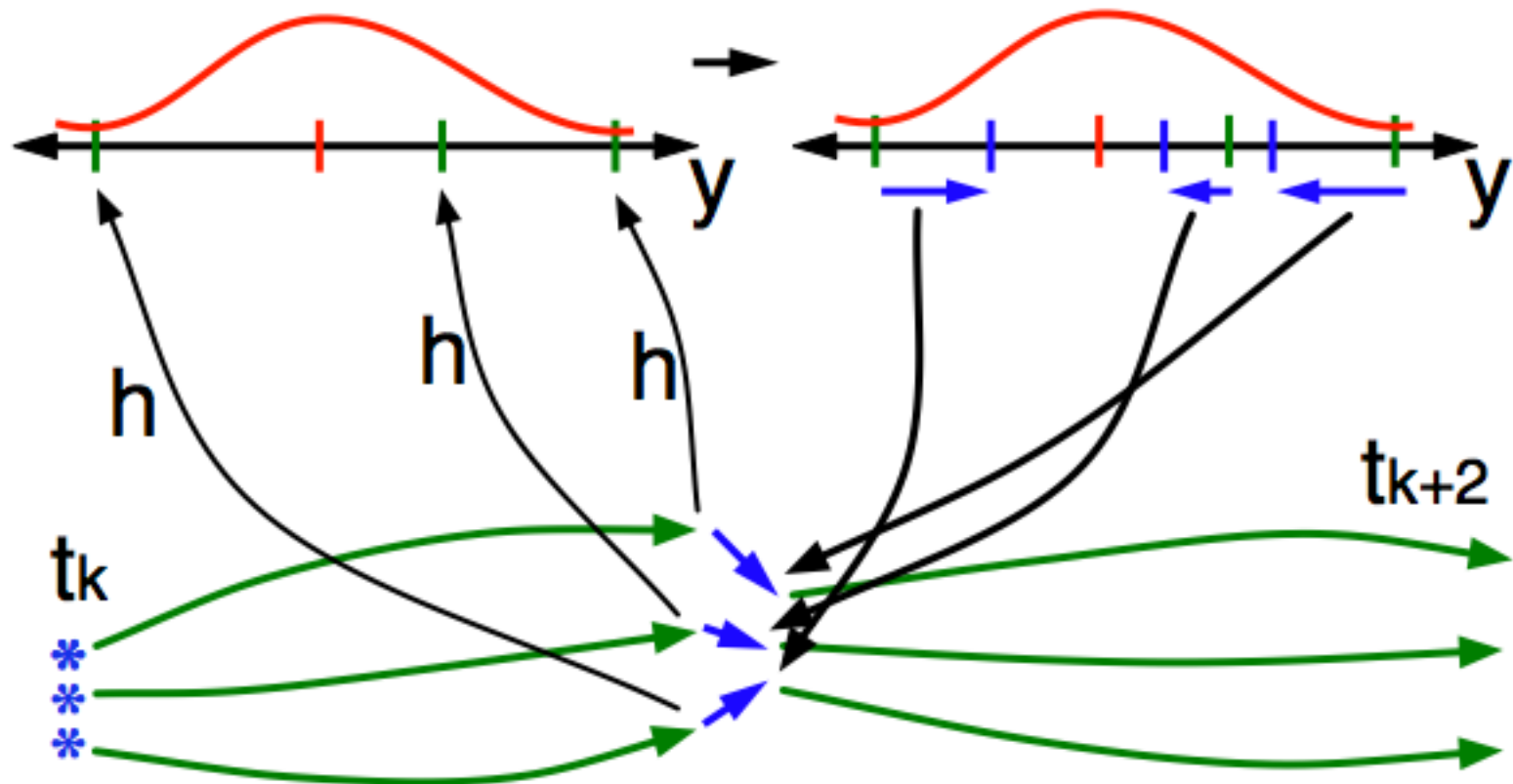
How an Ensemble Filter Works for Geophysical Data Assimilation

5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



How an Ensemble Filter Works for Geophysical Data Assimilation

- When all ensemble members for each state variable are updated, integrate to time of next observation ...

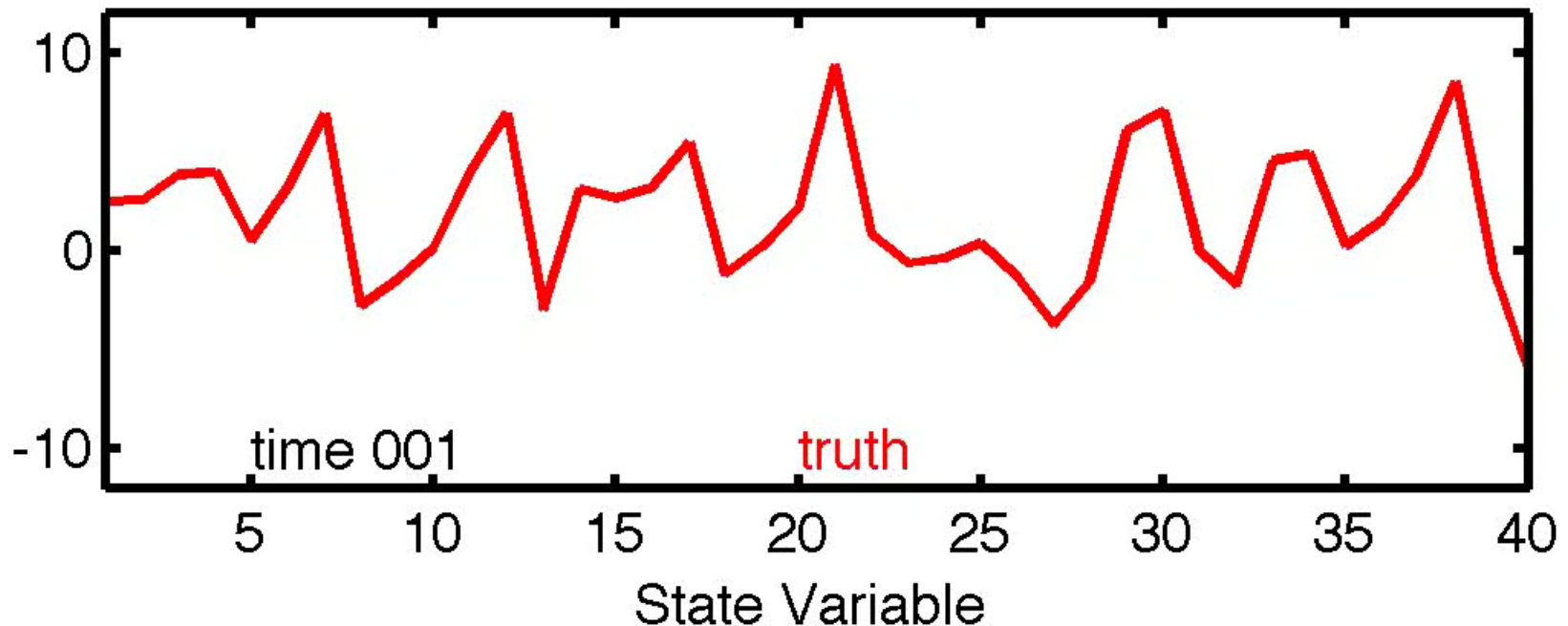


Ensemble Filter for Lorenz-96 40-Variable Model

40 state variables: X_1, X_2, \dots, X_{40} .

$$dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$$

Acts 'something' like weather around a latitude band.

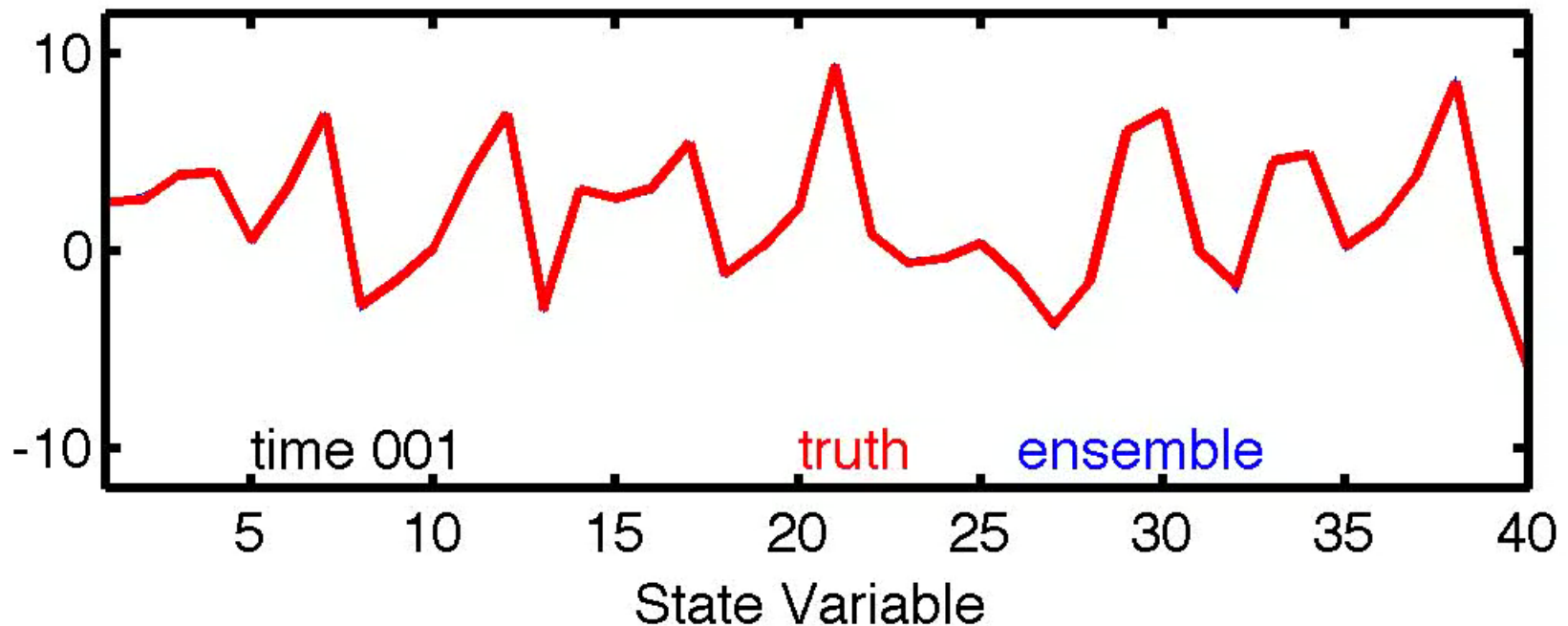


Lorenz-96 is sensitive to small perturbations

Introduce 20 'ensemble' state estimates.

Each is perturbed for each of the 40-variables at time 0.

Refer to unperturbed control integration as 'truth'.



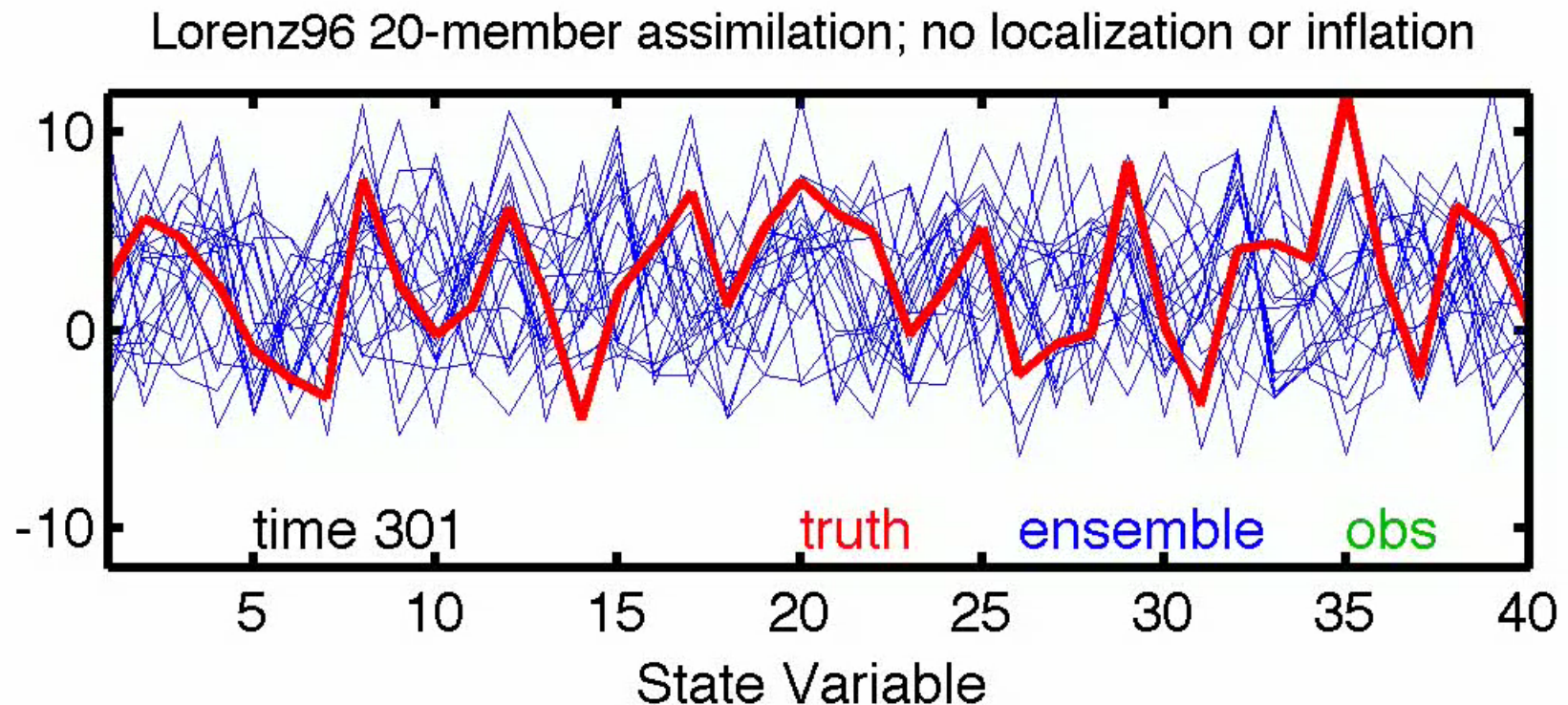
Assimilate 'observations' from 40 random locations.

Interpolate truth to station location.

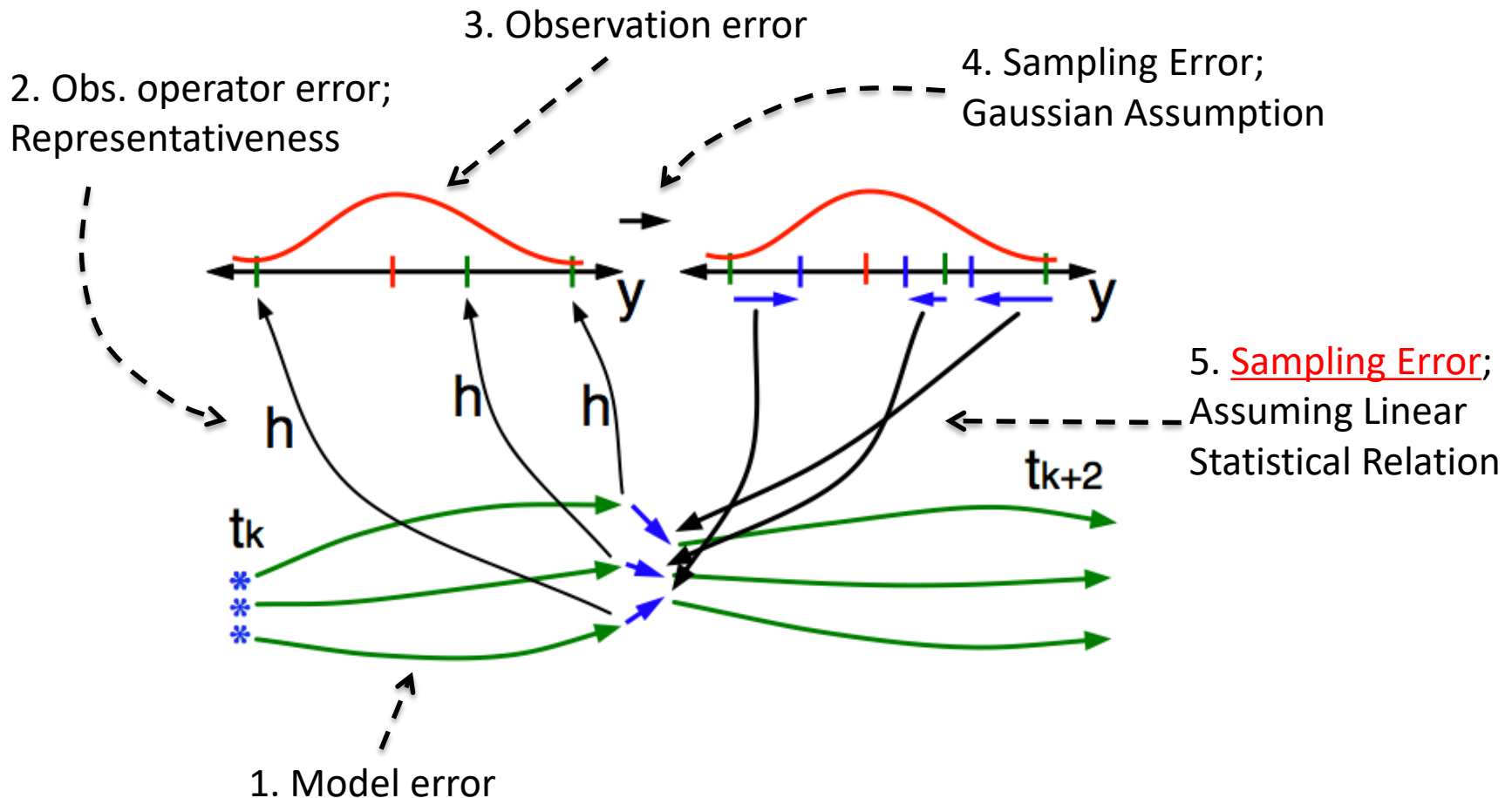
Simulate observational error:

Add random draw from $N(0, 16)$ to each.

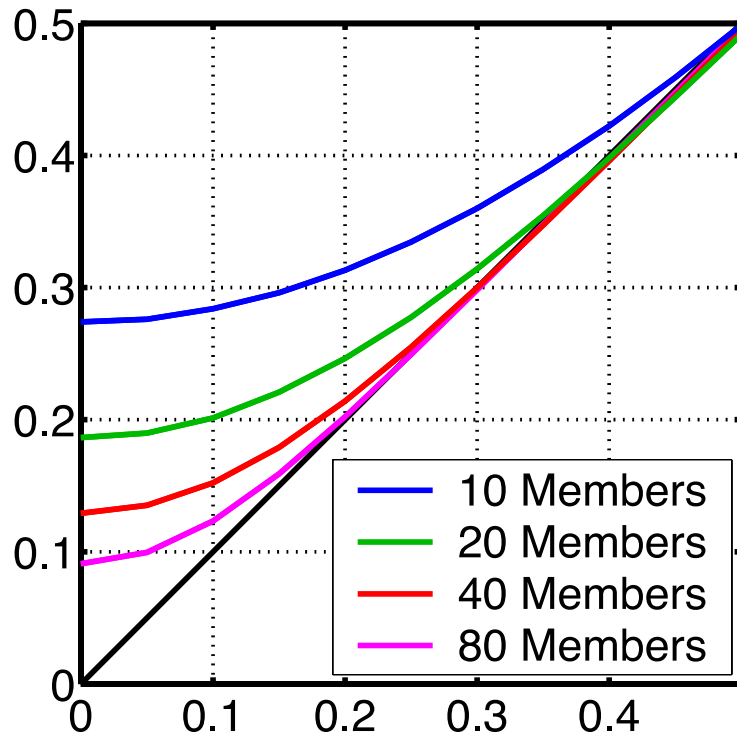
Start from 'climatological' 20-member ensemble.



Some Error Sources in Ensemble Filters



Sampling Error: Observations Impact Unrelated State Variables



Plot shows expected absolute value of sample correlation vs. true correlation.

Unrelated obs. reduce spread, increase error.

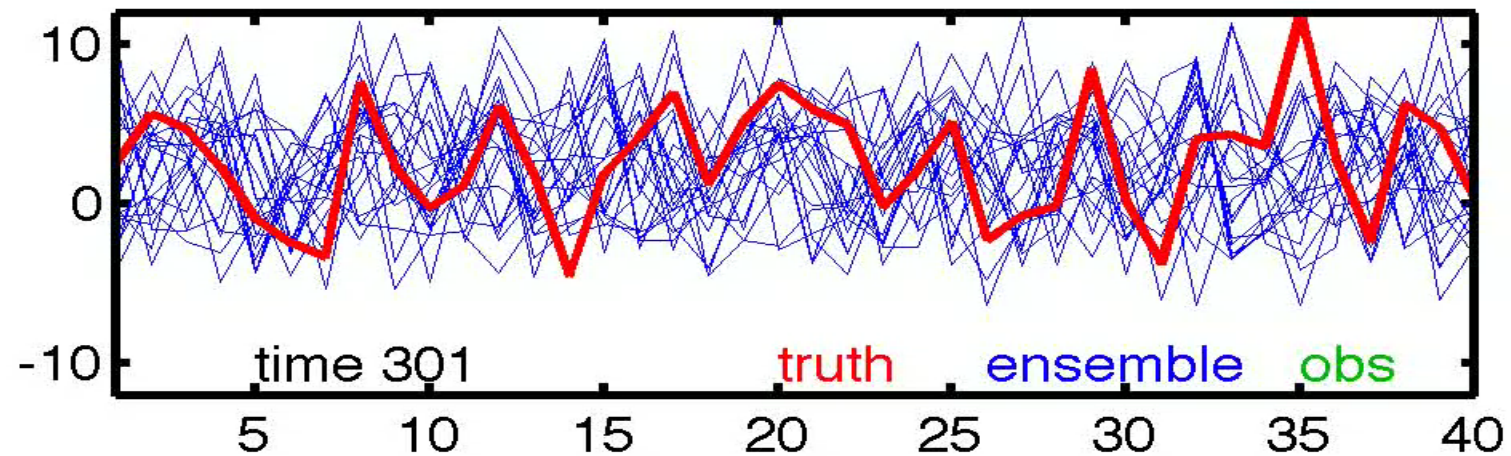
Attack with localization.

Reduce impact of observation on weakly correlated state variables.

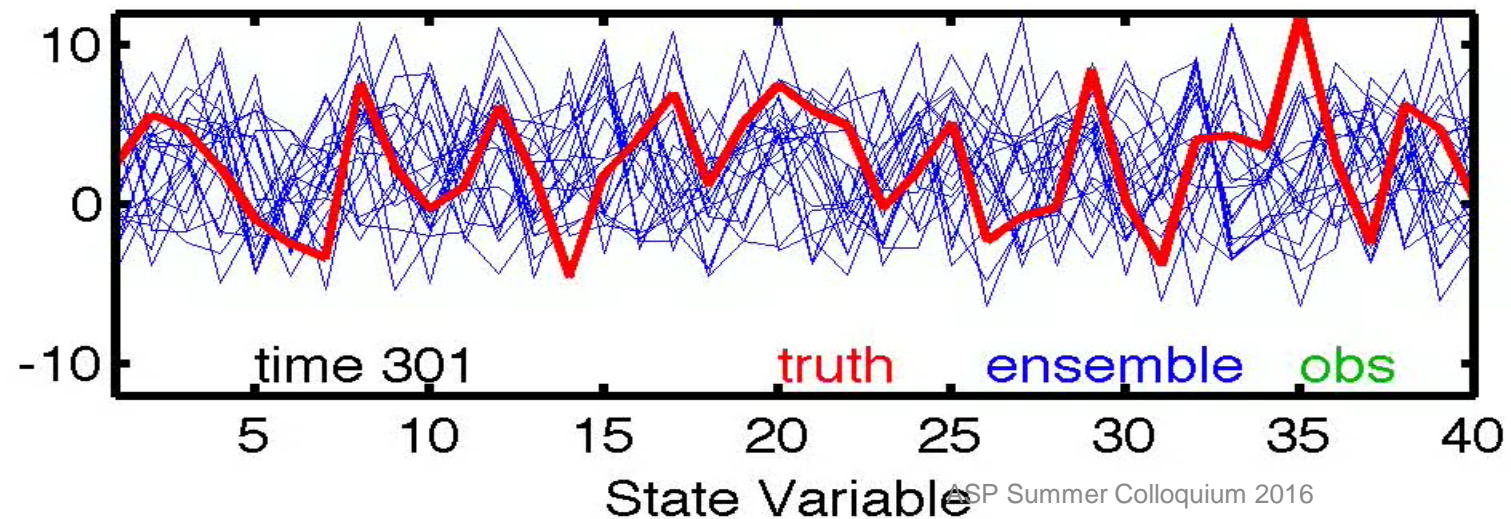
Let weight go to zero for many 'unrelated' variables to save on computing.

Lorenz-96 Assimilation with localization of observation impact

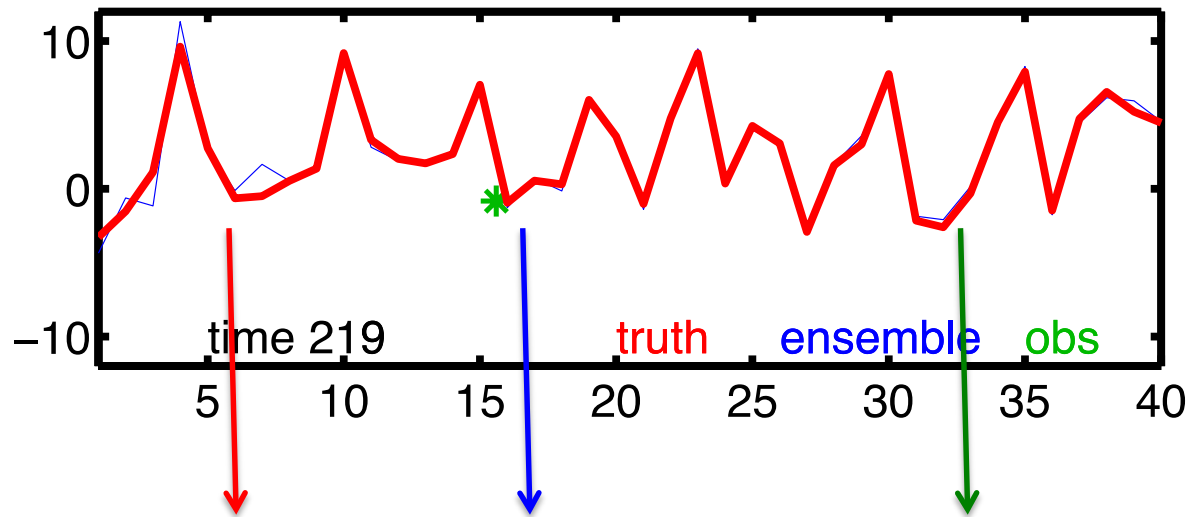
Localization from Hierarchical Filter



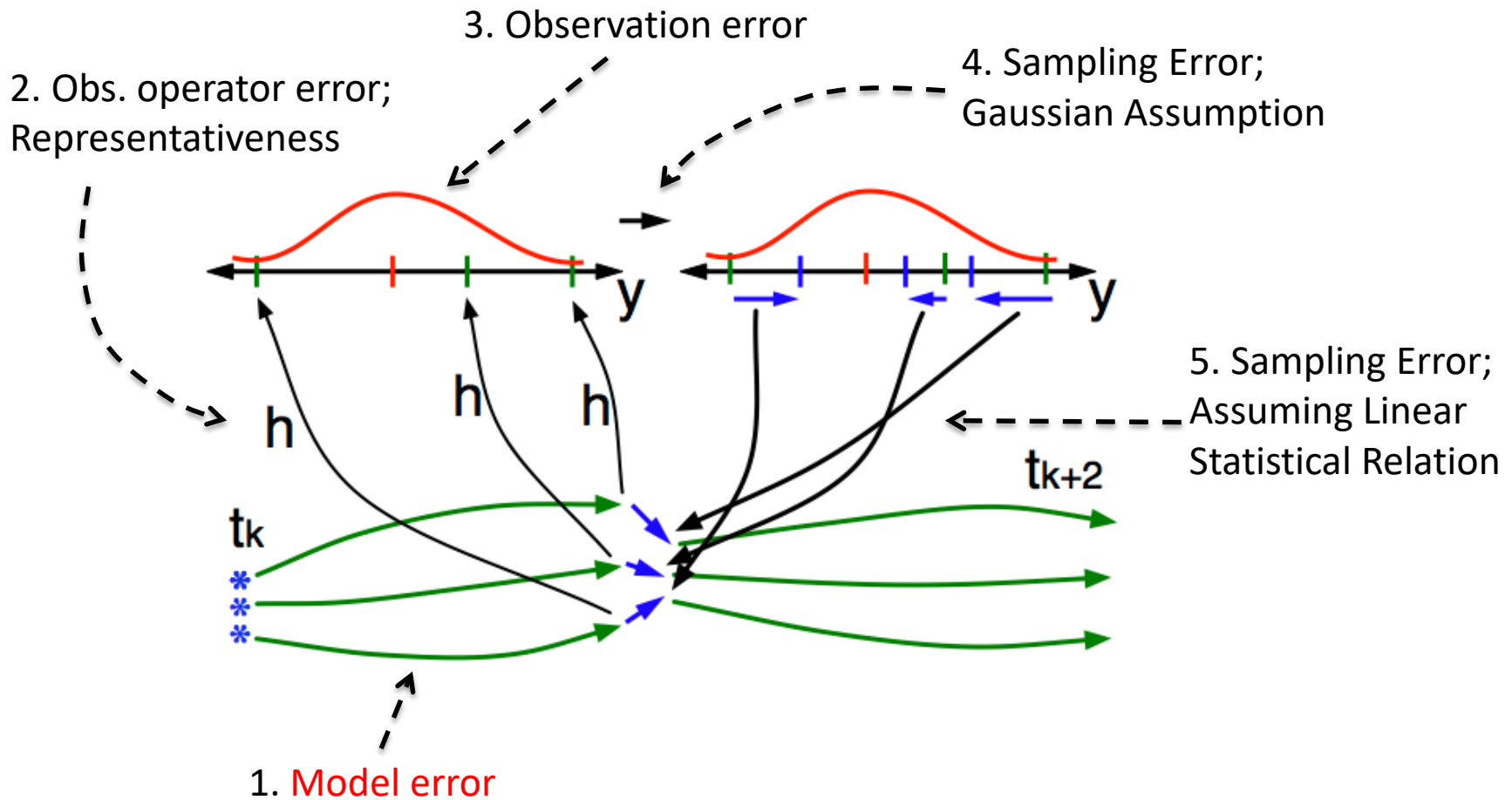
No Localization



Lorenz-96 Assimilation with localization of observation impact



Some Error Sources in Ensemble Filters

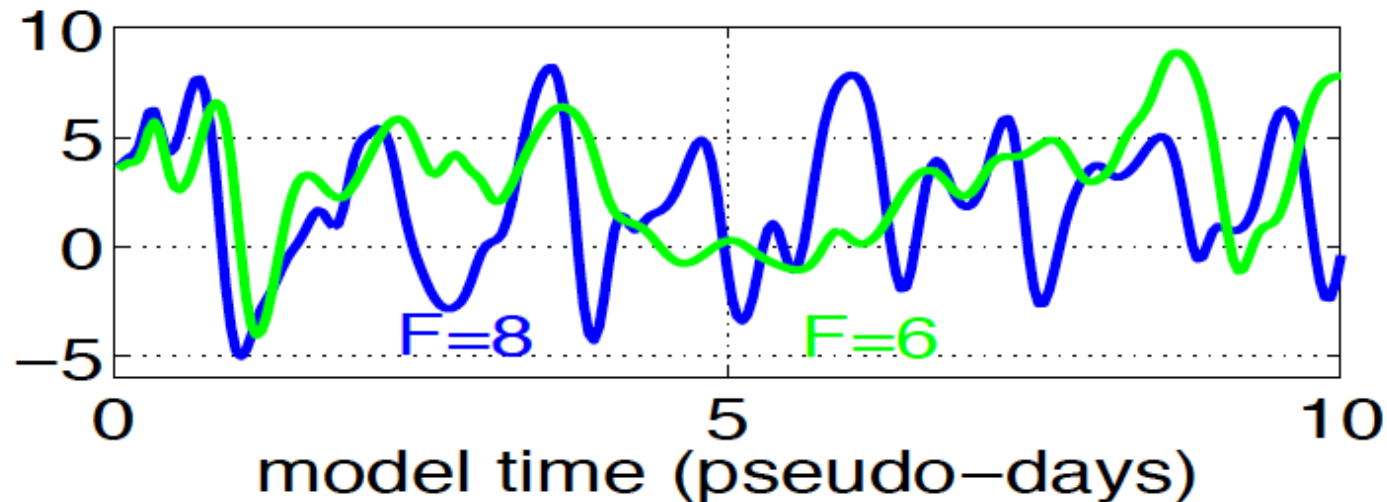


Assimilating in the presence of simulated model error

$$dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$$

For truth, use $F = 8$.

In assimilating model, use $F = 6$.



Time evolution for first state variable shown.

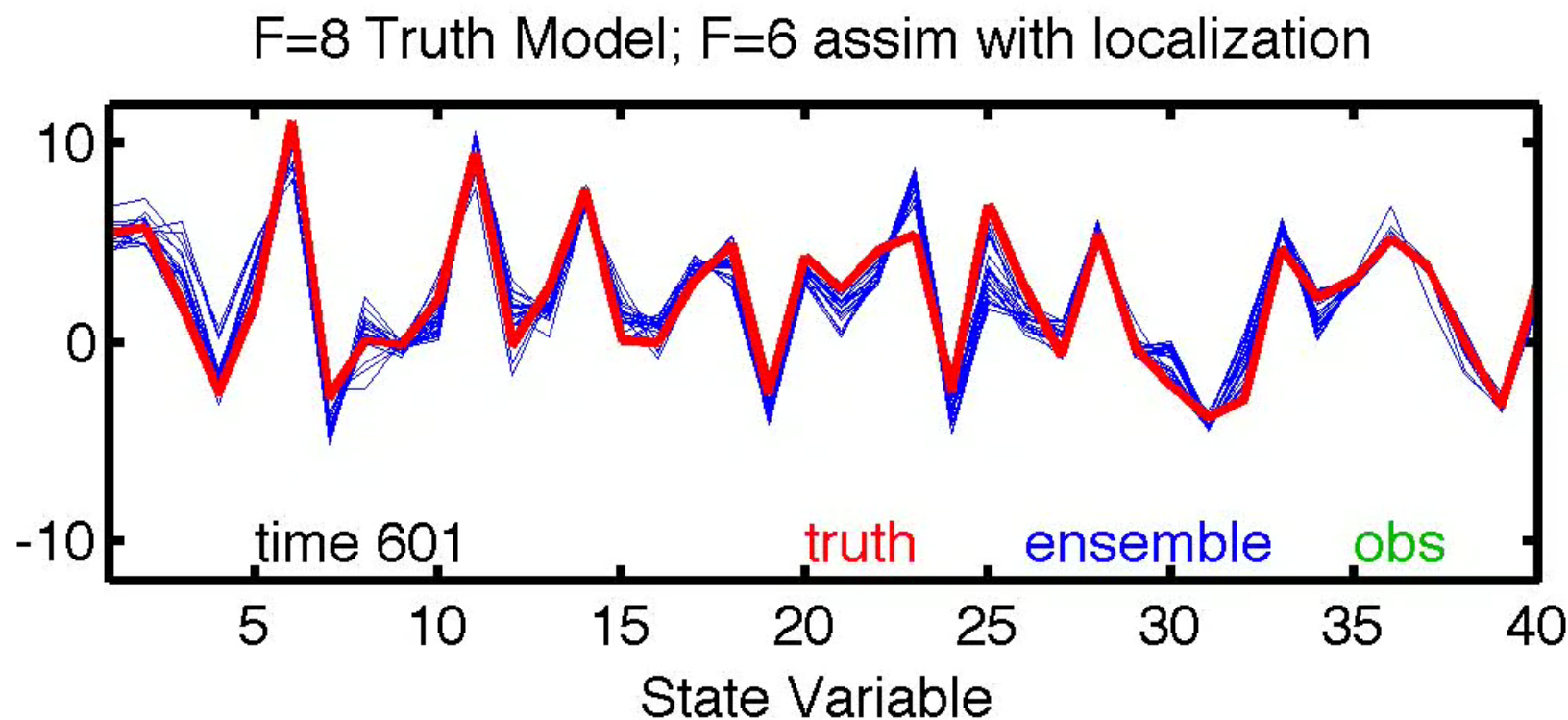
Assimilating model quickly diverges from 'true' model.

Assimilating in the presence of simulated model error

$$dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$$

For truth, use $F = 8$.

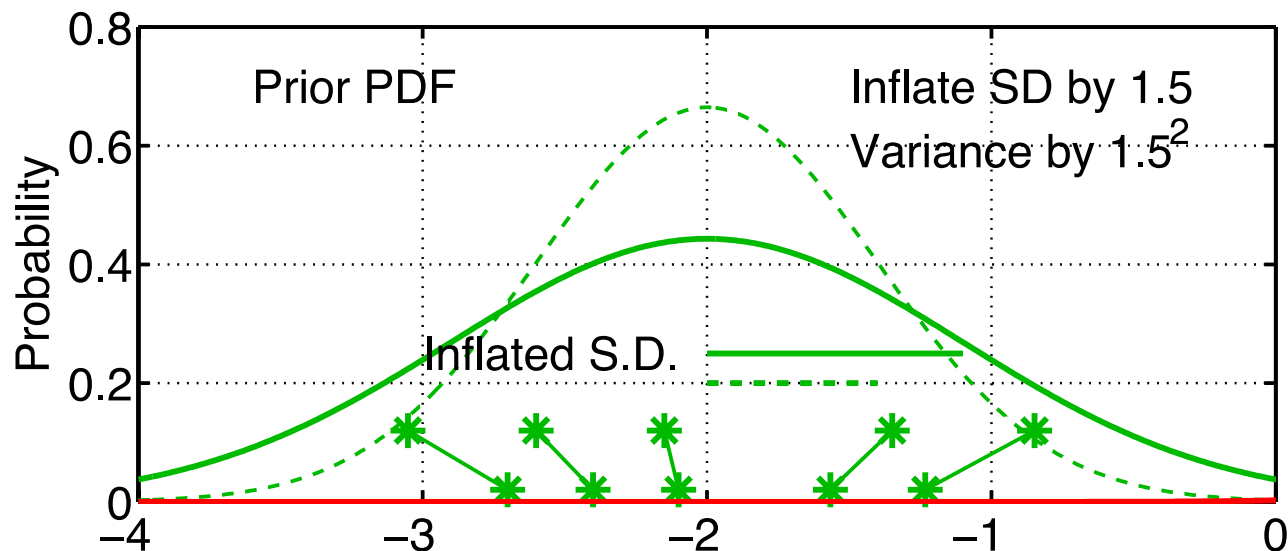
In assimilating model, use $F = 6$.



Reduce confidence in prior to deal with model error

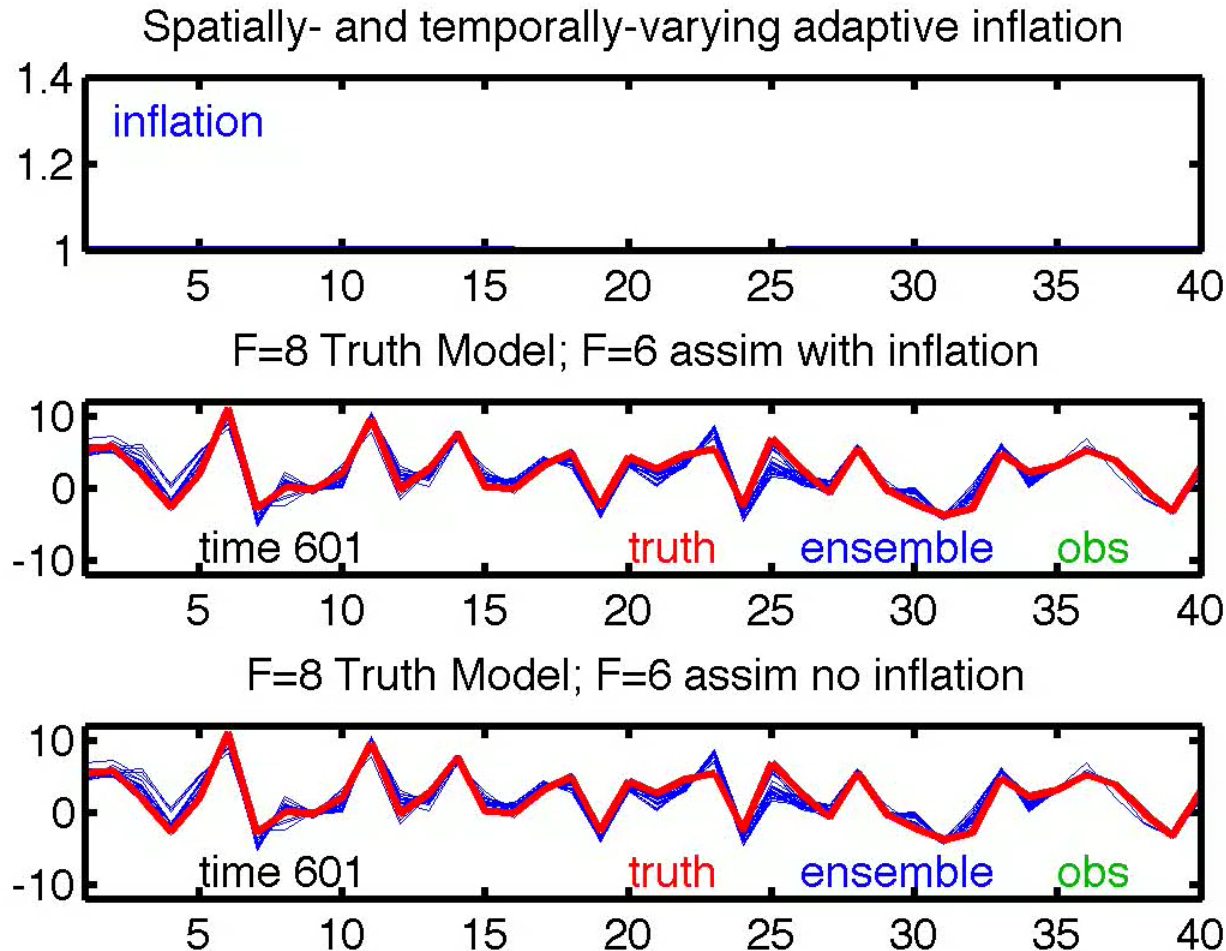
Use inflation.

Simply increase prior ensemble variance for each state variable.
Adaptive algorithms use observations to guide this.



Assimilating with Inflation in Presence of Model Error

Inflation is a function of state variable and time.
Automatically selected by adaptive inflation algorithm.

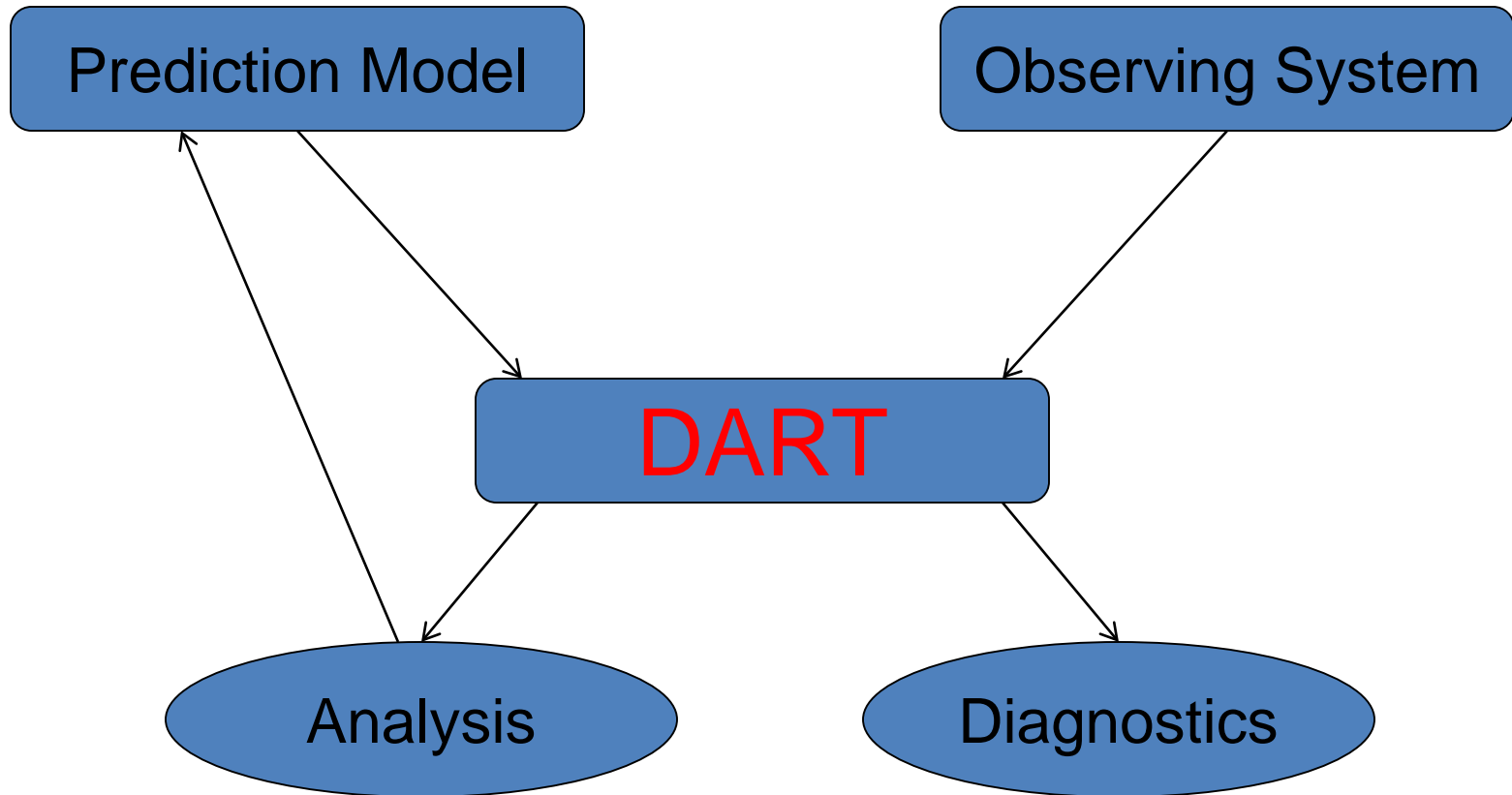


Uncertainty Quantification with an Ensemble Kalman Filter

- (Ensemble) KF optimal for linear model, gaussian likelihood, perfect model.
- In KF, only mean and covariance have meaning.
- Ensemble allows computation of many other statistics.
- What do they mean? Not entirely clear.
- What do they mean when there are all sorts of error?
Even less clear.
- Must Calibrate and Validate results.

The Data Assimilation Research Testbed (DART)

DART provides data assimilation 'glue' to build ensemble forecast systems for the atmosphere, ocean, land, ...



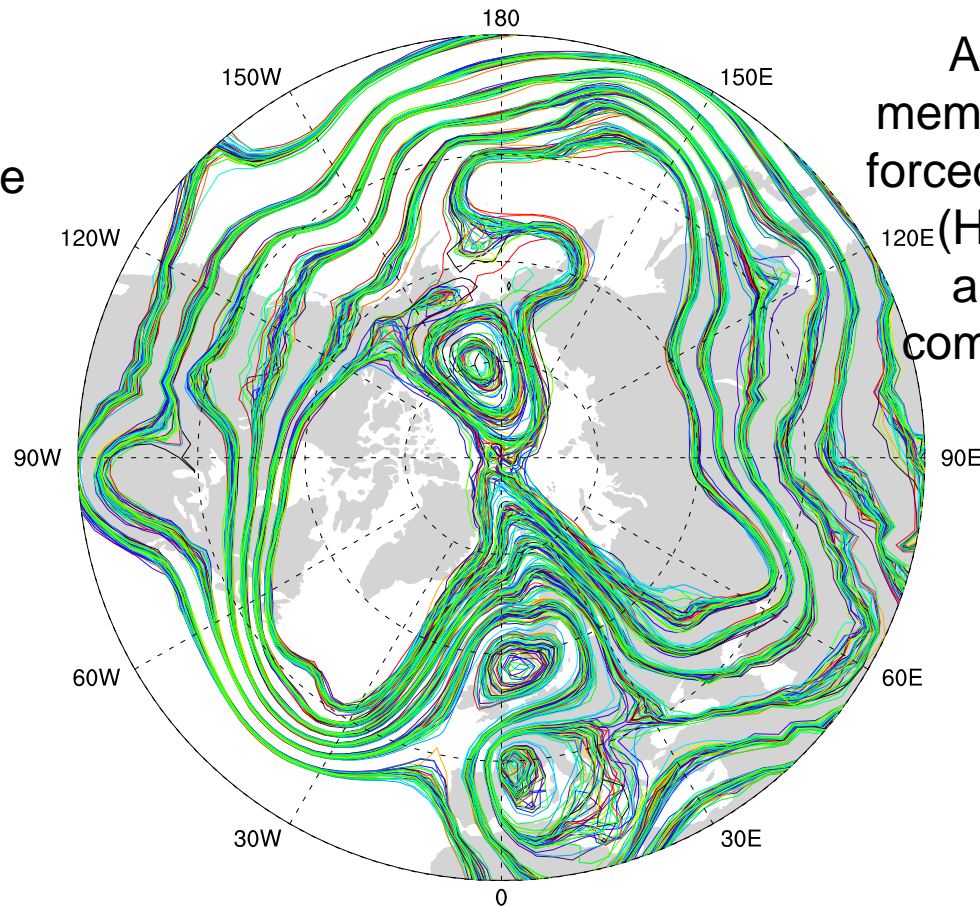
DART Science and Collaborators (1)

Science: A global atmospheric ensemble reanalysis.

Collaborators: Model Developers at NCAR

O(1 million)
atmospheric obs are
assimilated every
day.

500 hPa GPH
Feb 17 2003



Assimilation uses 80
members of 2° FV CAM
forced by a single ocean
(Hadley+ NCEP-OI2)
and produces a very
competitive reanalysis.

1998-2010
4x daily
is available.

Science: *Do new satellite observations of cloud motion improve hurricane forecasts?*

Atmospheric motion vectors from CIMMS at University of Wisconsin.

Collaborator: Ting-Chi Wu,
Graduate Student,
University of Miami.

DART Science and Collaborators (2)

Tropical Cyclones and Atmospheric Motion Vectors

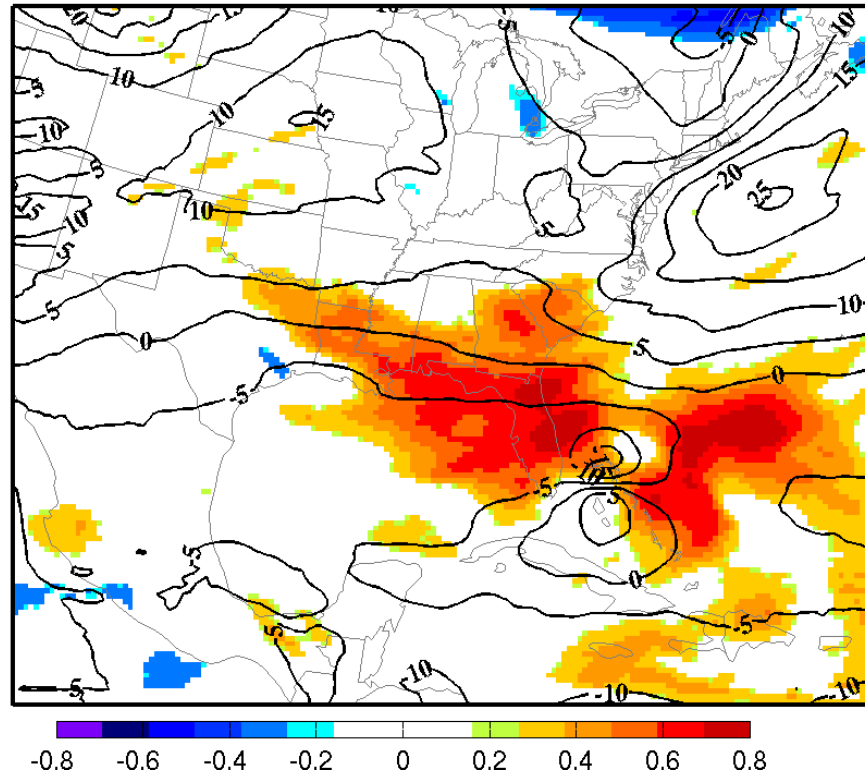
Wu et al., 2014, MWR, **142**, 49–71.

Science: *Where should more observations be taken to improve landfall forecasts?*

Ensemble sensitivity analysis for Katrina.

Collaborator: Ryan Torn, University at Albany.

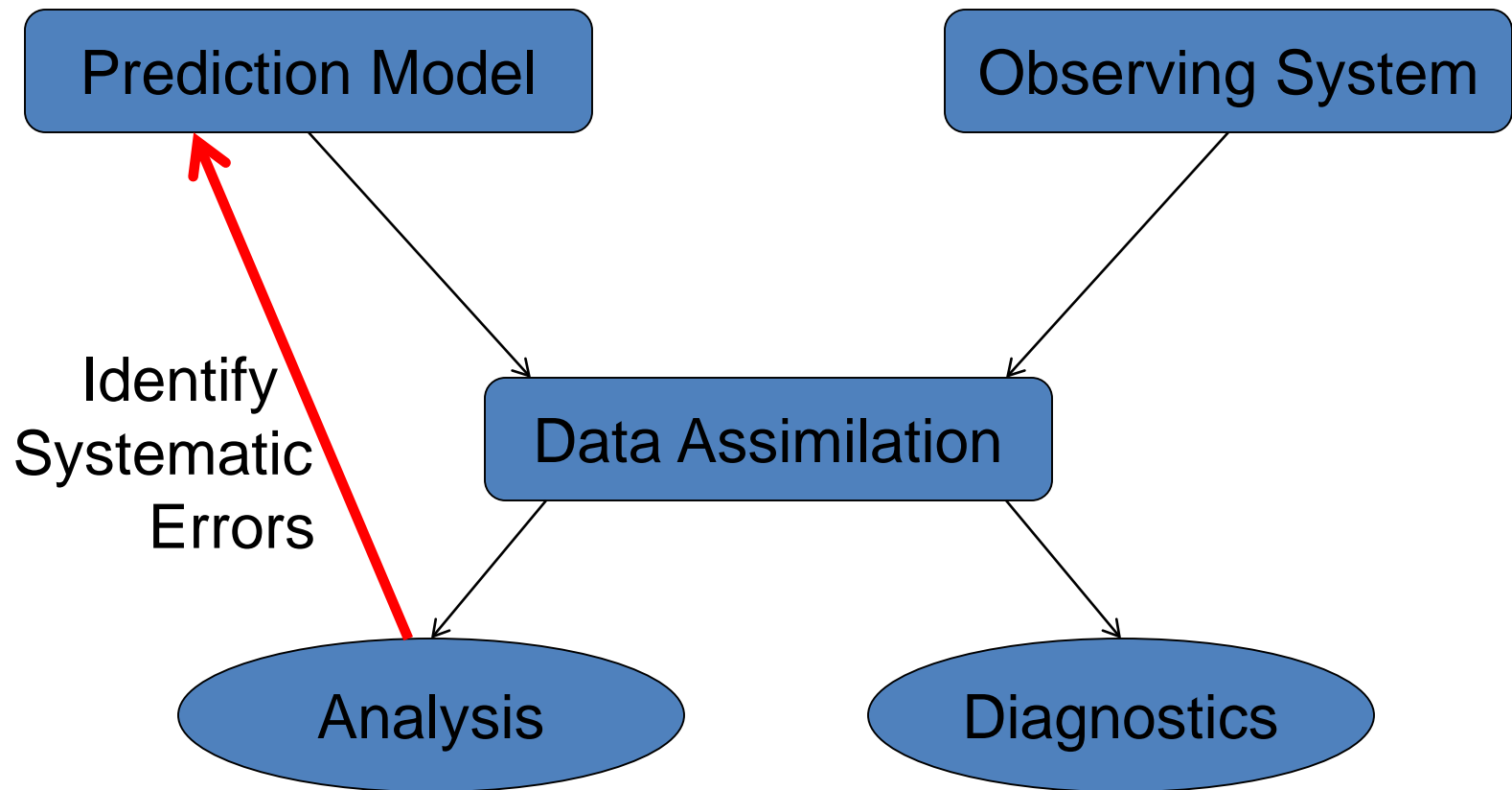
Hurricane Katrina Sensitivity Analysis



Contours are ensemble mean 48h forecast of deep-layer mean wind.

Color shows where observations could help.

Identifying Model Systematic Errors



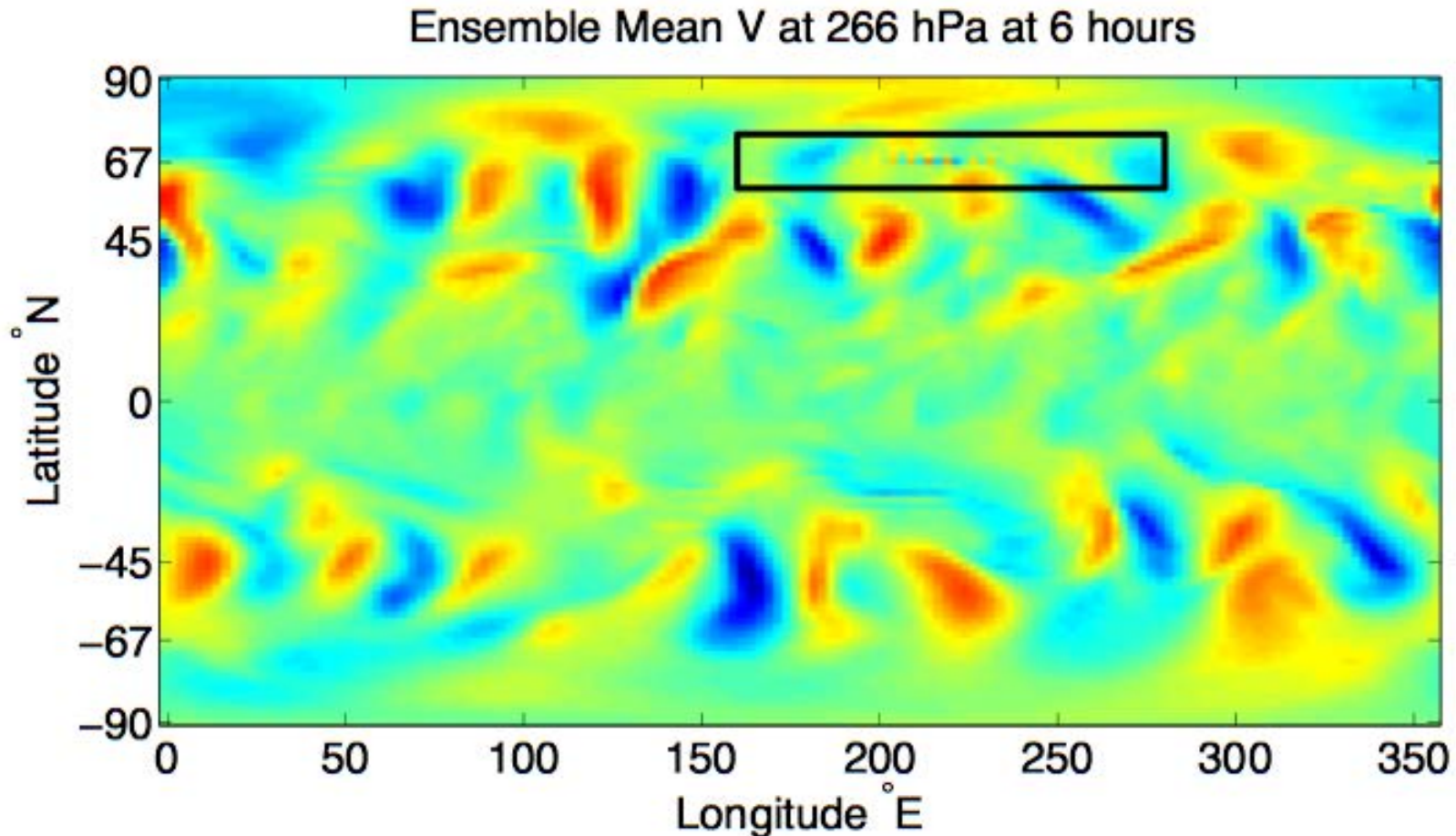
DART Science and Collaborators (4)

Science: Diagnosing and correcting errors in the CAM
FV core.

Collaborator: Peter Lauritzen, CGD.

DART Science and Collaborators (4)

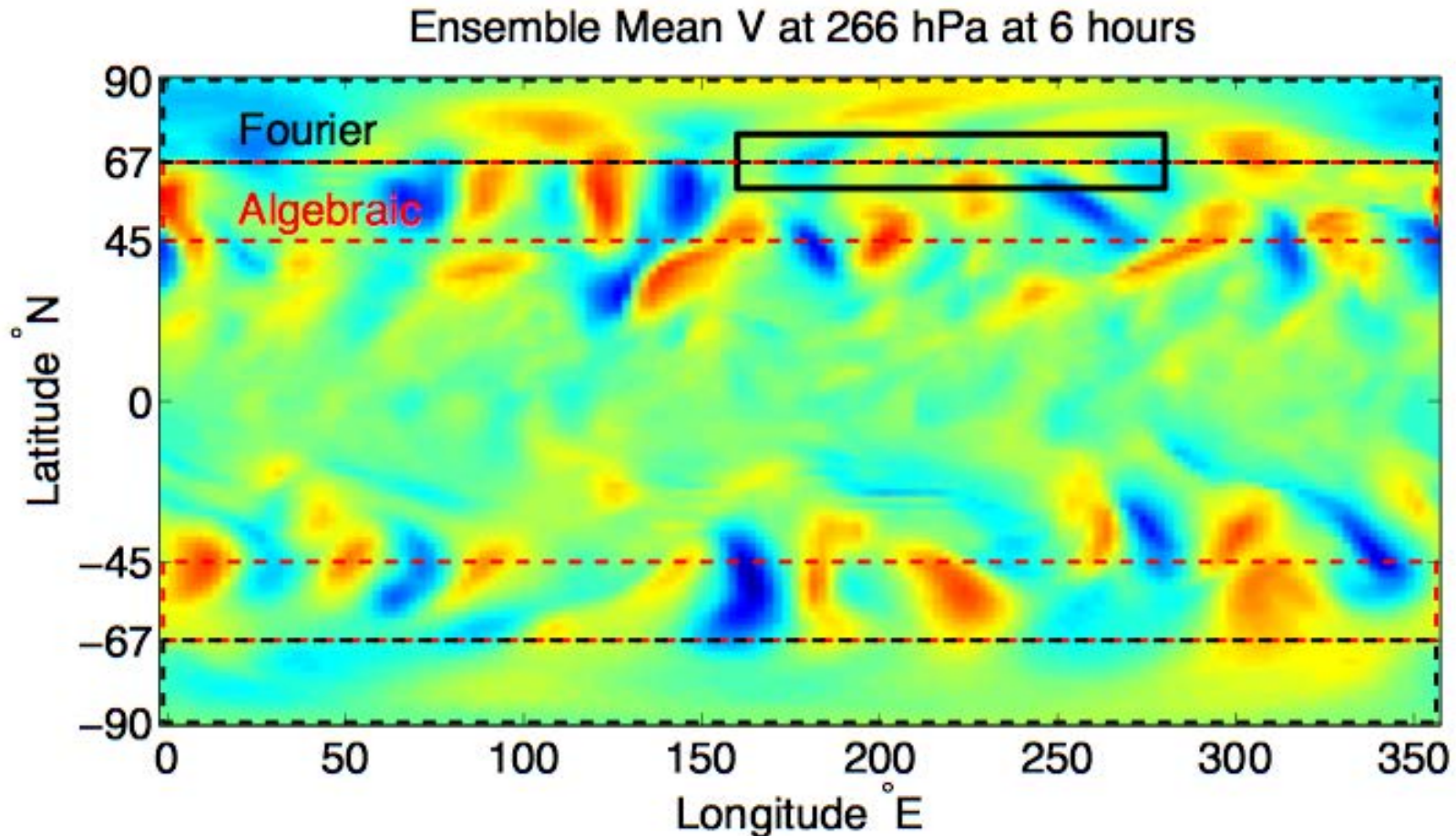
Gridpoint noise detected in CAM/DART analysis



CAM FV core - 80 member mean - 00Z 25 September 2006

DART Science and Collaborators (4)

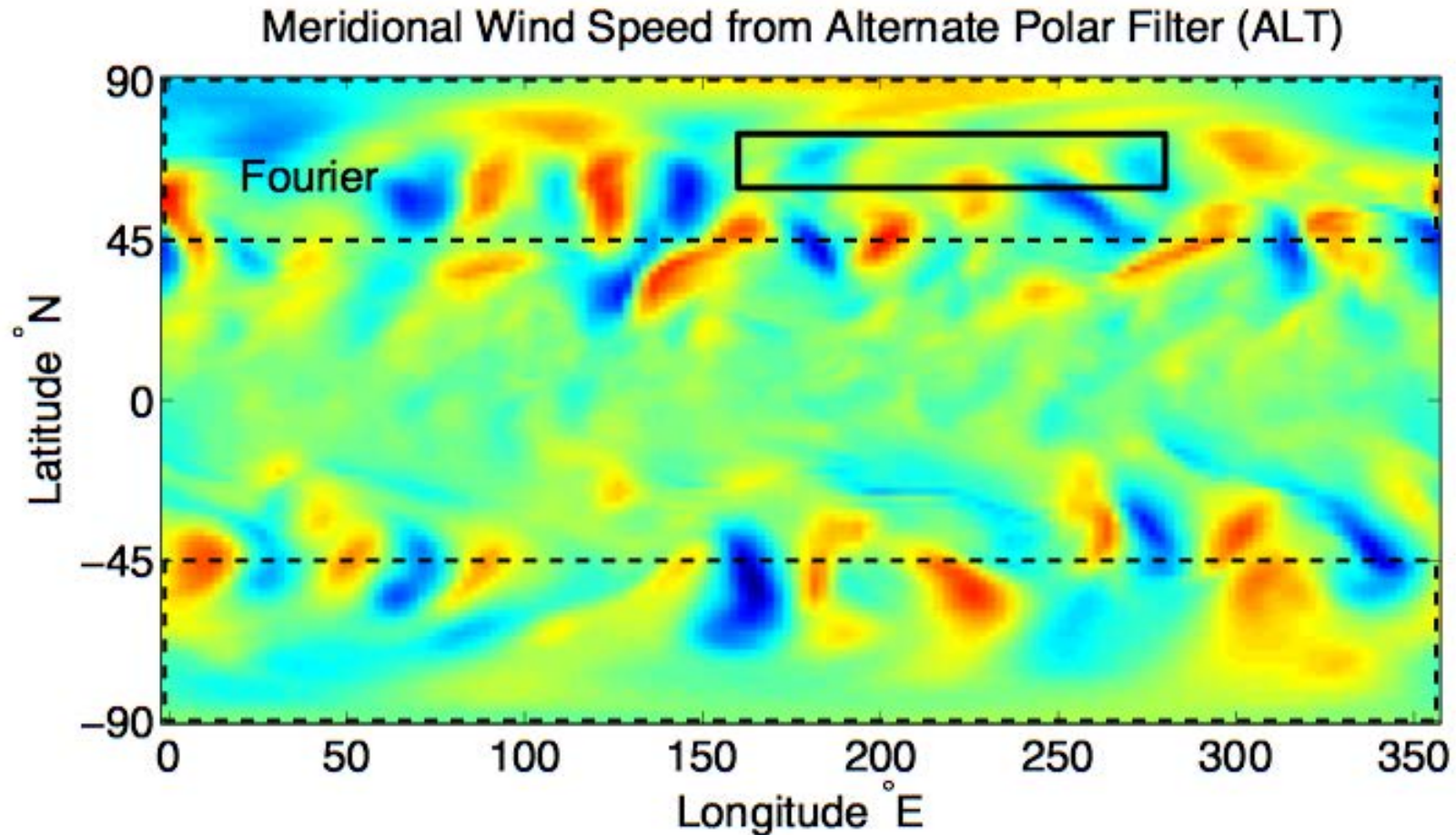
Suspensions turned to the polar filter (DPF)



CAM FV core - 80 member mean - 00Z 25 September 2006

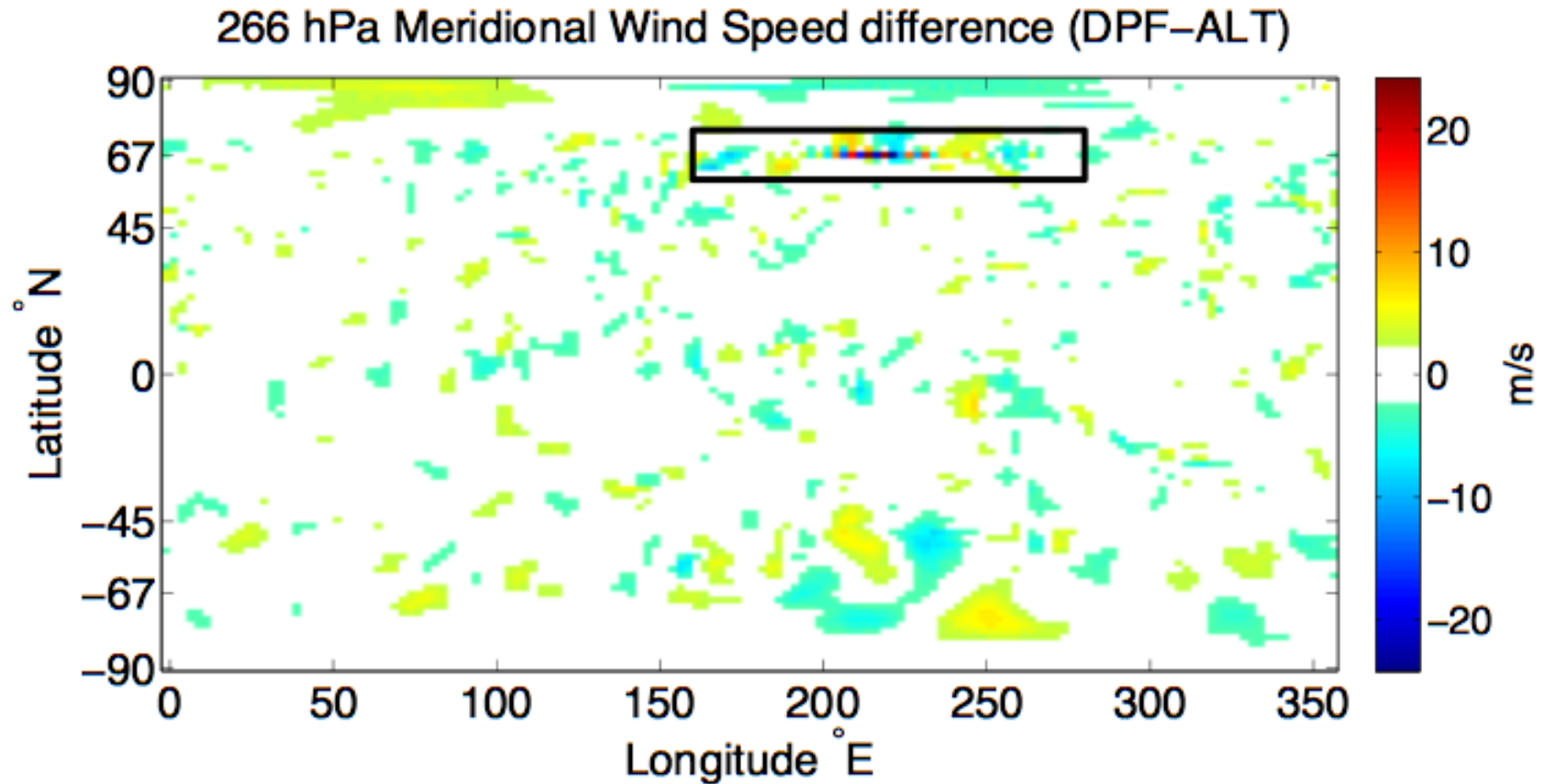
DART Science and Collaborators (4)

Continuous polar filter (alt-pft) eliminated noise.



DART Science and Collaborators (4)

Differences mostly in transition region of default filter.



- The use of DART diagnosed a problem that had been unrecognized (or at least undocumented).
- Could have an important effect on any physics in which meridional mixing is important.
- The problem can be seen in ‘free runs’ - it is not a data assimilation artifact.
- Without assimilation, can't get reproducing occurrences to diagnose.

DART Science and Collaborators (5)

Science: Global Ocean data assimilation.

Collaborators: Alicia Karspeck, Steve Yeager, CGD.

- Climate change over time scales of 1 to several decades has been identified as very important for mitigation and infrastructure planning.
- Need ocean initial conditions for the IPCC decadal prediction program (and maybe a crystal ball, too!).

DART Science and Collaborators (5)

World Ocean Database T, S observation counts.

These counts are for 1998 & 1999 and are representative.

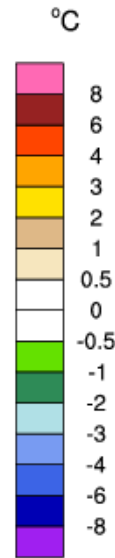
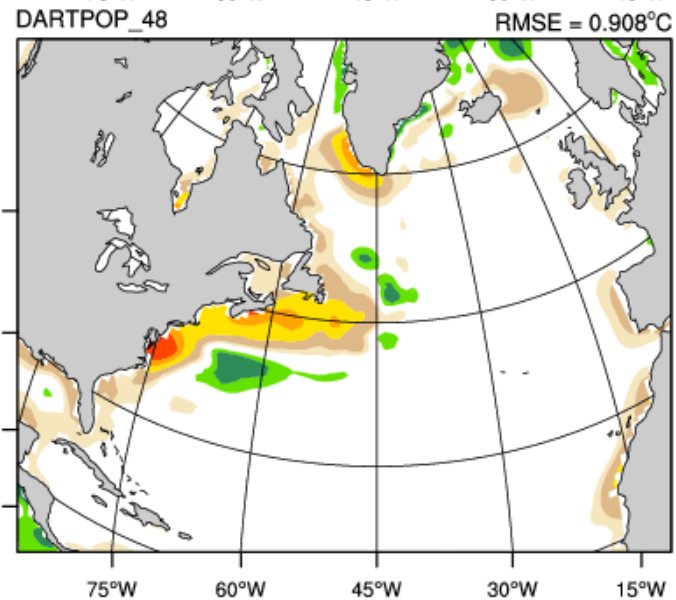
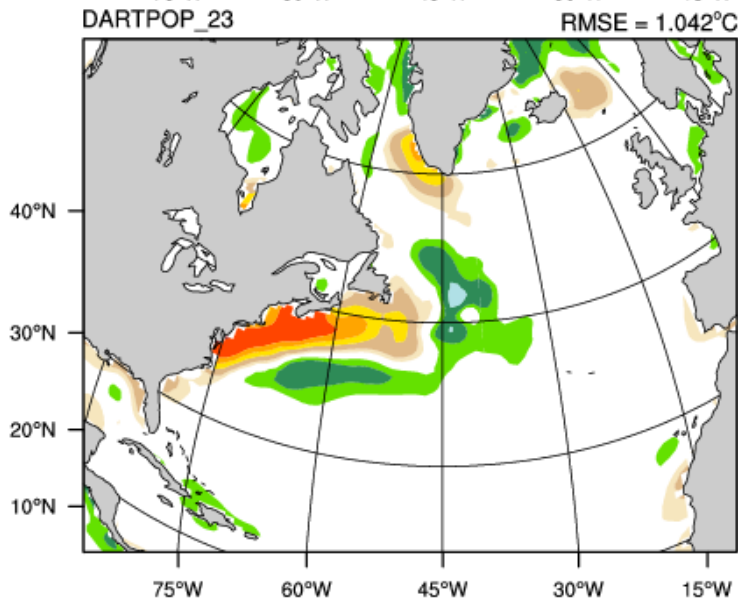
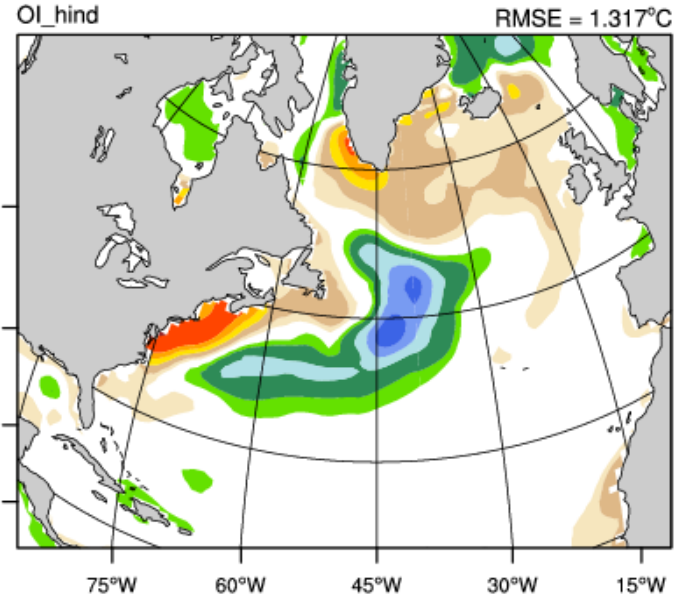
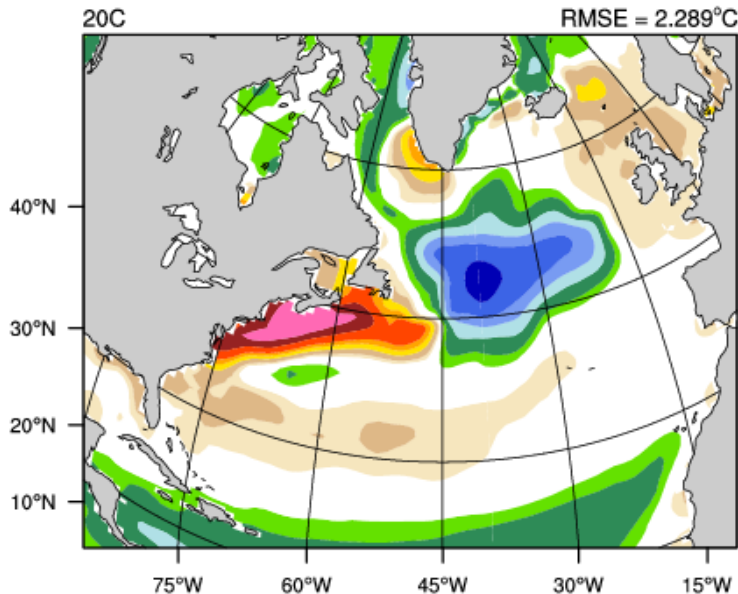
FLOAT_SALINITY	68200
FLOAT_TEMPERATURE	395032
DRIFTER_TEMPERATURE	33963
MOORING_SALINITY	27476
MOORING_TEMPERATURE	623967
BOTTLE_SALINITY	79855
BOTTLE_TEMPERATURE	81488
CTD_SALINITY	328812
CTD_TEMPERATURE	368715
STD_SALINITY	674
STD_TEMPERATURE	677
XCTD_SALINITY	3328
XCTD_TEMPERATURE	5790
MBT_TEMPERATURE	58206
XBT_TEMPERATURE	1093330
APB_TEMPERATURE	580111



Physical Space: 1998/1999 SST Anomaly from HadOI-SST

Coupled Free Run

23 POP 1 DATM



POP forced by observed
atmosphere (hindcast)

48 POP 48 CAM

DART Science and Collaborators (6)

Science: Land surface analysis with DART/CLM.

Collaborator: Yongfei Zhang, UT Austin.

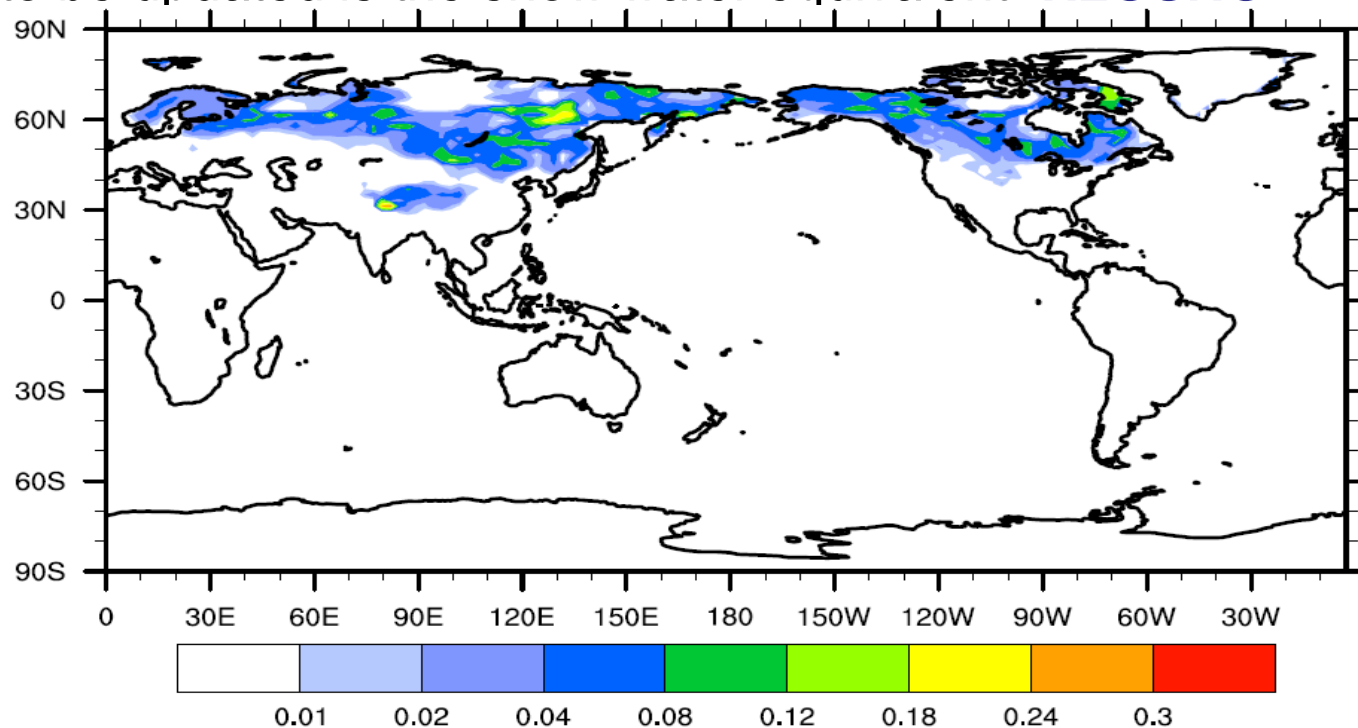
Land surface analysis with DART/CLM:

Estimate snow water equivalent with observations of snow cover fraction from satellites (MODIS).

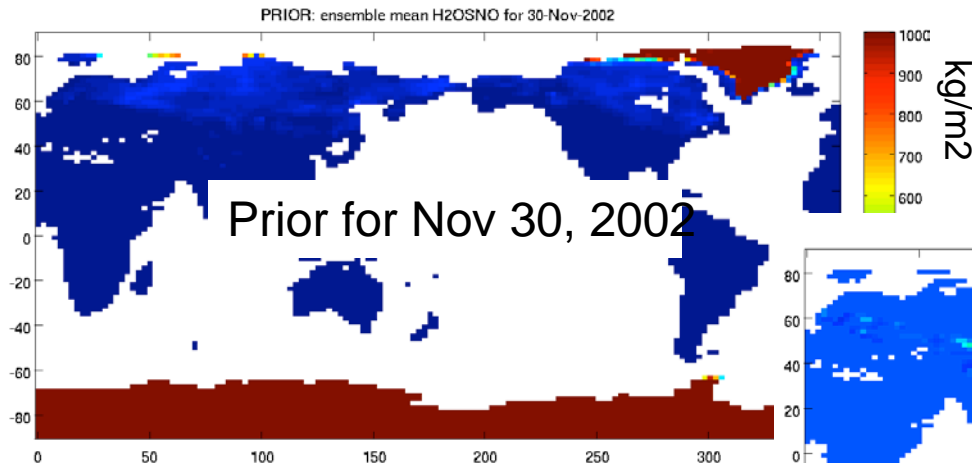
Assimilation of MODIS snow cover fraction

- 80 member ensemble for onset of NH winter
- Assimilate once per day
- Level 3 MODIS product – regridded to a daily 1 degree grid
- Observation error variance is 0.1 (for lack of a better value)
- Observations can impact state variables within 200km
- CLM variable to be updated is the snow water equivalent “**H2OSNO**”

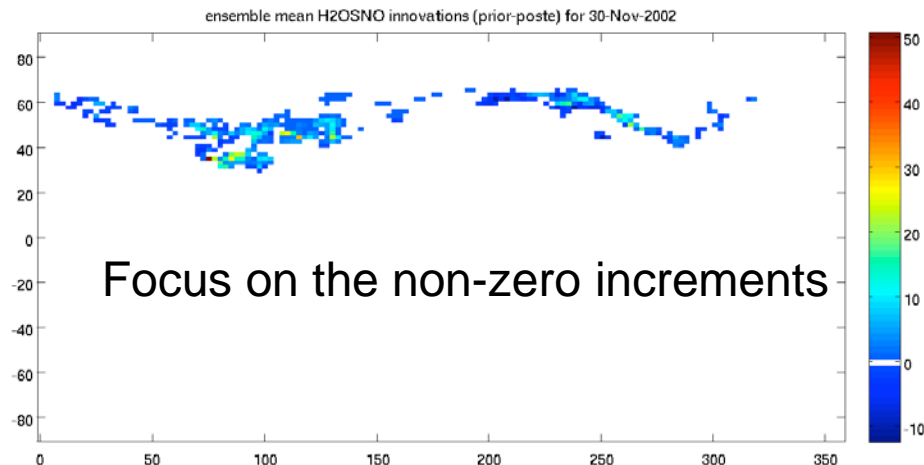
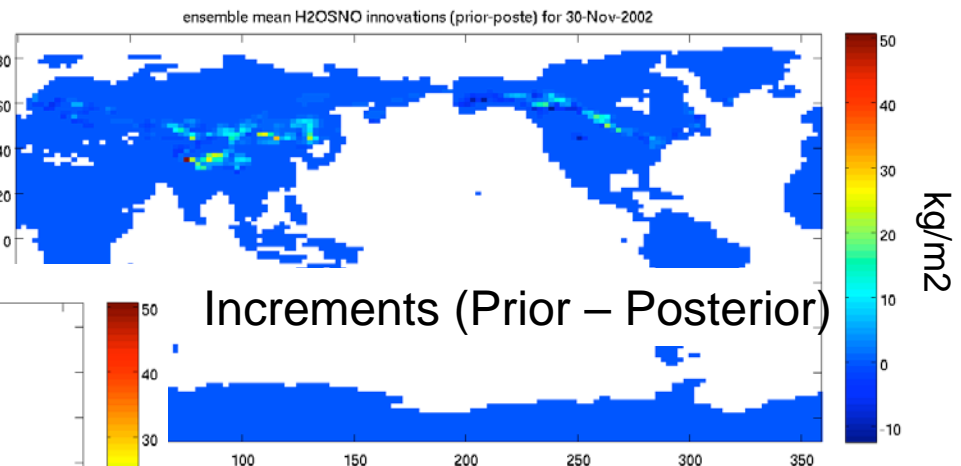
Standard
deviation of the
snow cover
fraction initial
conditions for
Oct. 2002



An early result: assimilation of MODIS snowcover fraction on total *snow water equivalent* in CLM.



Thanks Yongfei!



The model state is changing in reasonable places, by reasonable amounts. At this point, that's all we're looking for.

DART Science and Collaborators (7)

Science: Regional Atmospheric Chemistry.

Collaborator: Arthur Mizzi, NCAR/ACD.

- **WRF-Chem** – Weather Research and Forecasting Model (WRF) with online chemistry.
- **Meteorological Observations** – NOAA PREPBUFR conventional observations.
- **Chemistry Observations** – MOPITT CO retrieval profiles (also IASI CO retrievals – results not shown).

WRF/Chem Chemical Weather Forecast System

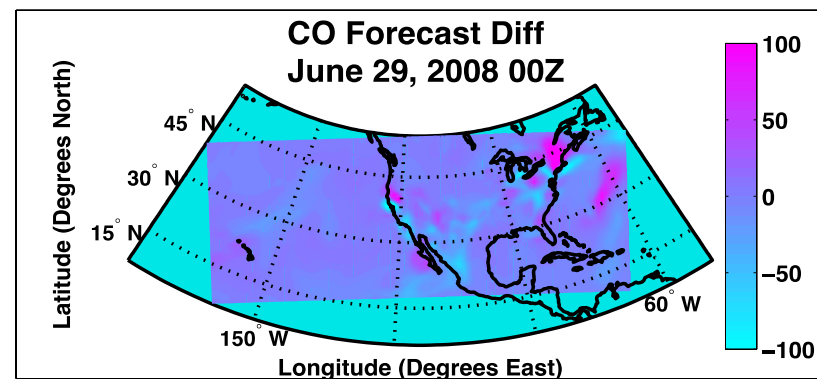
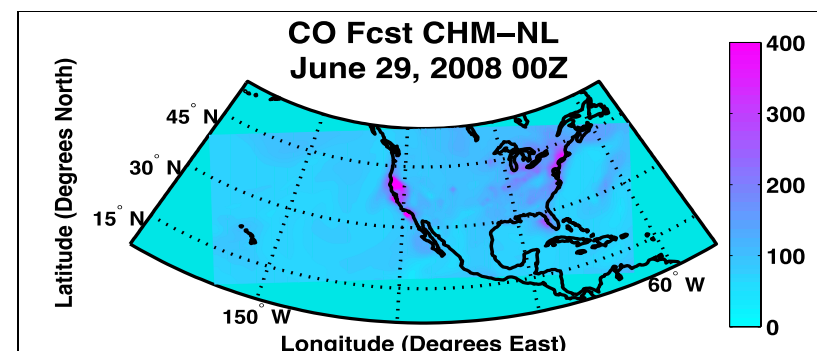
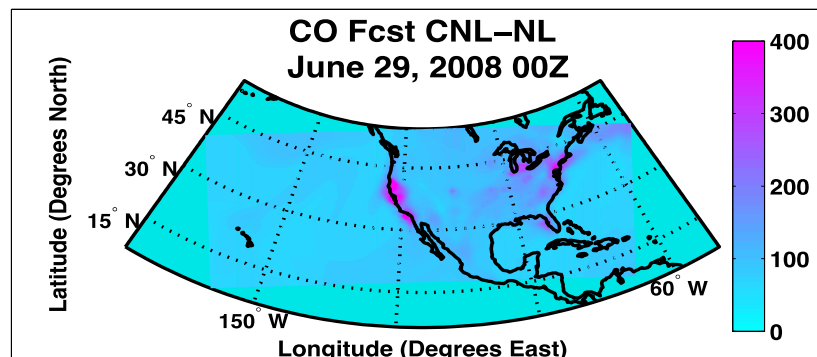
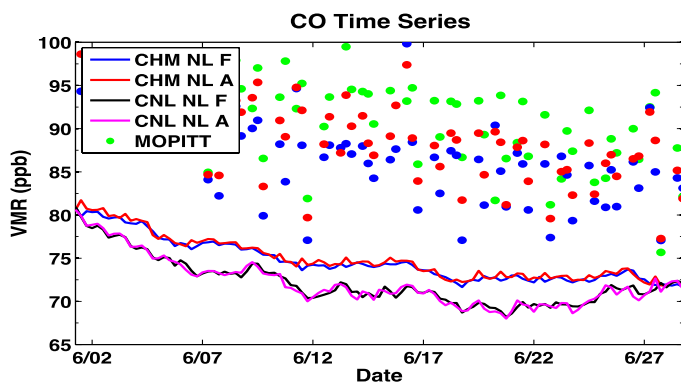
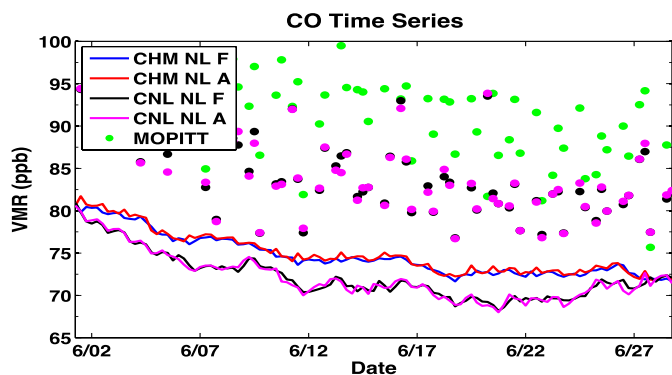
- **WRF/Chem-DART** cycling with conventional meteorological observations and MOPITT CO V5 retrieval profiles.
- Continuous six-hr cycling (00Z, 06Z, 12Z, and 18Z).
- CONUS grid with 101x41x34 grid points and 100 km resolution.
- 20-member ensemble.
- June 1 - 30, 2008 (112 cycles) study period.
- Full state variable/obs interaction.
- Initial and lateral chemical boundary conditions from MOZART-4 simulation.
- Emissions: Biogenic – MEGAN, Anthropogenic – global inventories, and Fire – Fire Inventory from NCAR (FINN).

WRF/Chem Chemical Weather Forecast System

➤ Two experiments:

- ✧ Exp 1: PREPBUFR conventional obs (**CNTL DA**).
- ✧ Exp 2: MOPITT CO retrieval profiles and PREPBUFR conventional obs (**CHEM DA**).

WRF/Chem Chemical Weather Forecast System



DART Science and Collaborators (8)

Science: Global Atmospheric Chemistry.

Collaborators: Jerome Barre,

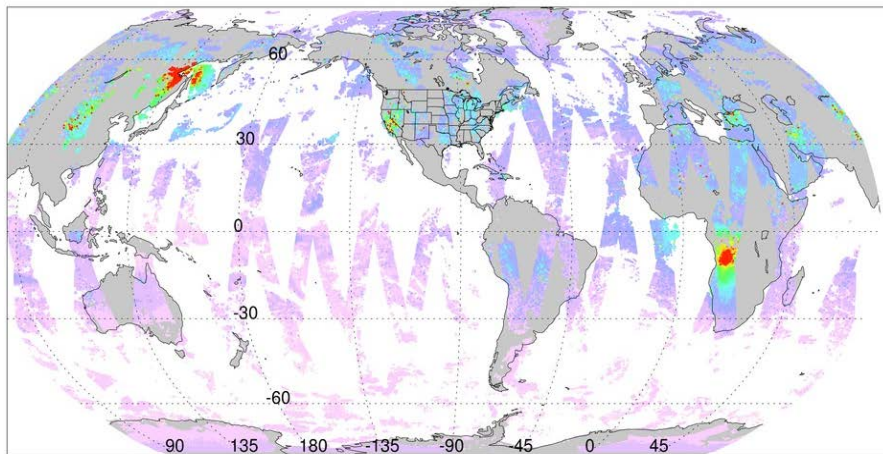
Benjamin Gaubert, NCAR/ACD.

Uses global CAM/Chem model, 1 degree.

Have full meteorological assimilation capability already.

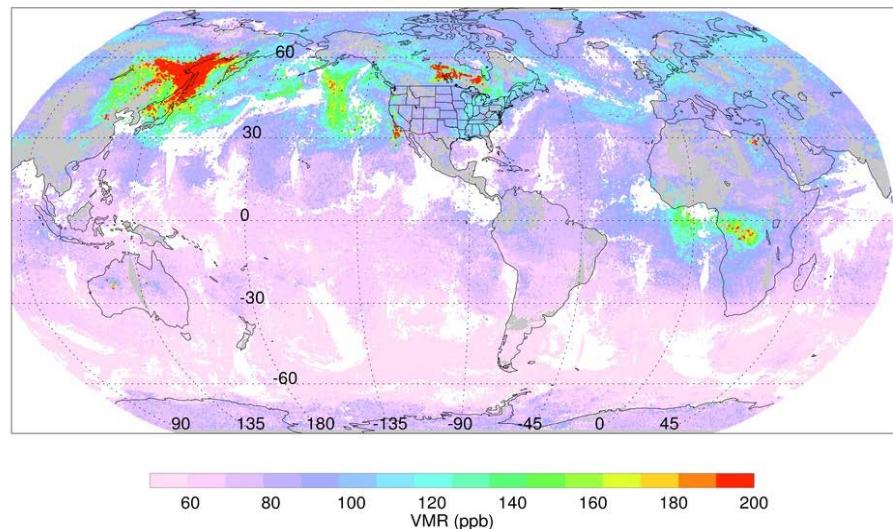
CAM/Chem Chemical DA System

a) NCAR/ACD MOPITT CO Total Column Effective VMR 1 Jul 2008



MOPITT CO:
On TERRA satellite
tropospheric profiles
Global coverage in 4 days
Multispectral retrievals
high sensitivity on surface land/day

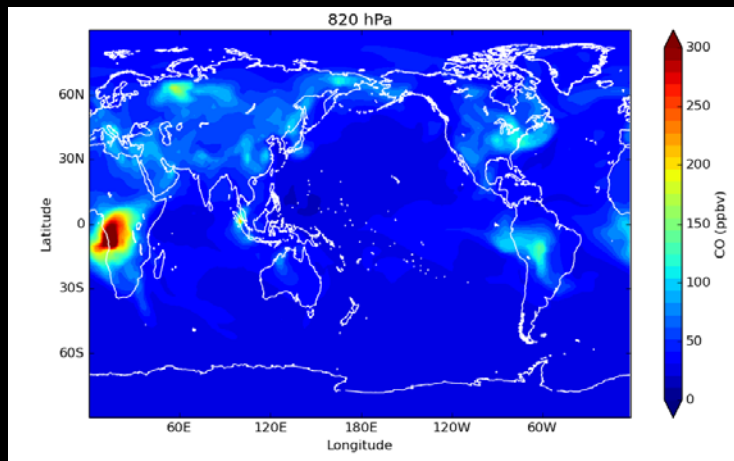
b) NCAR/FORLI IASI CO Total Column Effective VMR 1 Jul 2008



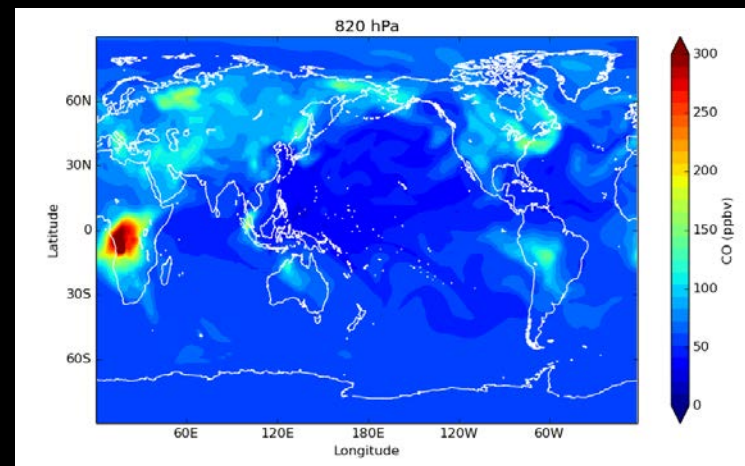
IASI CO:
On MetOpA satellite
tropospheric profiles
Global coverage in 1 day
Only thermal infrared
Sensitivity on upper PBL &
mid troposphere

CAM/Chem Chemical DA System

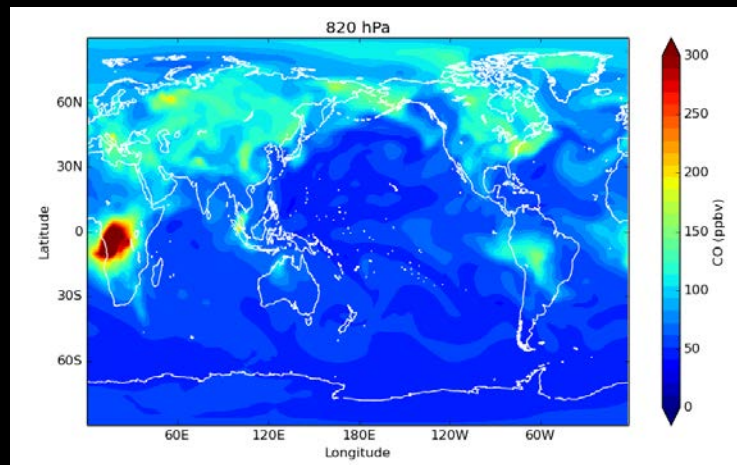
Control run: Met Only assimilated



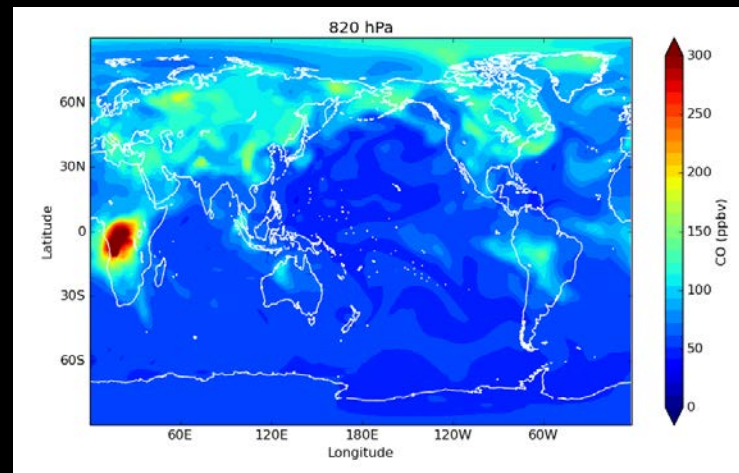
MOPITT run: Met + MOPITT assimilated



IASI run: Met + IASI assimilated



Combined run: Met + MOP+ IASI assimilated



Parameter Estimation

A system governed by (stochastic) Difference Equation:

$$dx_t = f(x_t, t) + G(x_t, t) d\beta_t, \quad t \geq 0 \quad (1)$$

Observations at discrete times:

$$y_k = h(x_k, t_k) + v_k; \quad k = 1, 2, \dots; \quad t_{k+1} > t_k \geq t_0 \quad (2)$$

Observational error white in time and Gaussian (nice, not essential).

$$v_k \rightarrow N(0, R_k) \quad (3)$$

Complete history of observations is:

$$Y_\tau = \{y_l; t_l \leq \tau\} \quad (4)$$

Goal: Find probability distribution for state:

$$p(x, t | Y_t) \quad \text{Analysis} \quad p(x, t^+ | Y_t) \quad \text{Forecast} \quad (5)$$

Parameter Estimation

A system governed by (stochastic) Difference Equation:

$$dx_t = f(x_t, t) + G(x_t, t)d\beta_t, \quad t \geq 0 \quad (1)$$

Parameter Estimation

A system governed by (stochastic) Difference Equation:

$$dx_t = f(x_t, t; \alpha) + G(x_t, t) d\beta_t, \quad t \geq 0 \quad (1)$$

Most geophysical models have 'tuning' parameters.

Model prediction might also depend on 'external forcing'.

Example: Sources of chemical tracers.

Parameter Estimation

A system governed by (stochastic) Difference Equation:

$$dx_t = f(x_t, t; \alpha) + G(x_t, t) d\beta_t, \quad t \geq 0 \quad (1)$$

One solution: State augmentation.

Define extended state vector $x^+ = (x, \alpha)$

Prediction model becomes (just a change in notation):

$$dx^+_t = f(x^+_t, t) + G(x_t, t) d\beta_t, \quad t \geq 0$$

Parameter Estimation

Define extended state vector $x^+ = (x, \alpha)$

Prediction model becomes:

$$dx^+_t = f(x^+_t, t) + G(x_t, t) d\beta_t, \quad t \geq 0$$

Problem: In general, no time prediction model for parameters.

If we had a prediction model, they would just have been state.

Kalman filter prior covariance comes from prediction model.

Parameter Estimation

Define extended state vector $x^+ = (x, \alpha)$

Prediction model becomes:

$$dx^+_t = f(x^+_t, t) + G(x_t, t) d\beta_t, \quad t \geq 0$$

Prior ensembles for parameters must be specified.

The prior sample covariance controls the impact of observations on parameters.

If prior covariance is not well-known, estimating parameters can be challenging.

Learn more about DART at:



www.image.ucar.edu/DAReS/DART

dart@ucar.edu

Anderson, J., Hoar, T., Raeder, K., Liu, H., Collins, N., Torn, R., Arellano, A., 2009: *The Data Assimilation Research Testbed: A community facility*.

BAMS, **90**, 1283—1296, doi: 10.1175/2009BAMS2618.1

